# Floquet drives, inhomogeneous CFT, and diffeomorphism representations

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Archipelagic perspectives on mathematics, physics and perceptible spectra of reality

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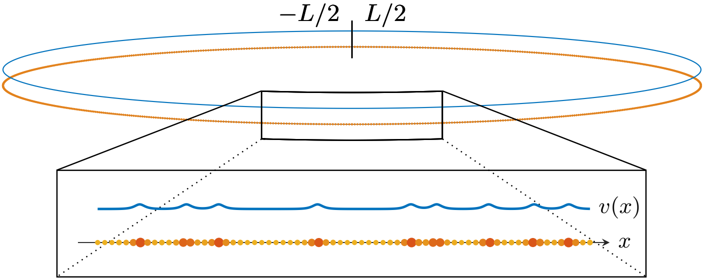
Based on joint works with K. Gawedzki, E. Langmann, and B. Lapierre

# Motivation: Inhomogeneous spin chain




P.M., arXiv:1912.04821, accepted in Ann. Henri Poincaré

Quantum $XXZ$ spin chain: $[S_j^\alpha, S_{j'}^\beta] = \mathrm{i}\delta_{j,j'}\epsilon_{\alpha\beta\gamma}S_j^\gamma$ and

$$H_{XXZ} = -\sum_{j=1}^{N} J_j \left( S_j^x S_{j+1}^x + S_j^y S_{j+1}^y - \Delta S_j^z S_{j+1}^z \right) - \sum_{j=1}^{N} h_j S_j^z$$

with $|\Delta| < 1$ and $h_j \propto J_j > 0$ varying on scales $\gg a$ and $\ll L = Na$.

# Inhomogeneous 1+1D CFT

Hamiltonian

$$H = \int_{-L/2}^{L/2} \mathrm{d}x \, v(x) \big[T_+(x) + T_-(x)\big]$$

with a smooth position-dependent velocity $v(x) = v(x+L) > 0$.

The operators $T_\pm(x) = T_\pm(x+L)$ satisfy

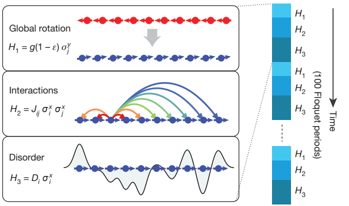
$$\big[T_\pm(x), T_\pm(y)\big] = \mp 2\mathrm{i}\delta'(x-y)T_\pm(y) \pm \mathrm{i}\delta(x-y)T'_\pm(y) \pm \frac{c}{24\pi}\mathrm{i}\delta'''(x-y),$$
$$\big[T_\pm(x), T_\mp(y)\big] = 0.$$

In Fourier space: Two commuting copies of the Virasoro algebra.

Energy-momentum tensor: $T_+ = T_{--}$, $T_- = T_{++}$, and $T_{+-} = 0 = T_{-+}$.

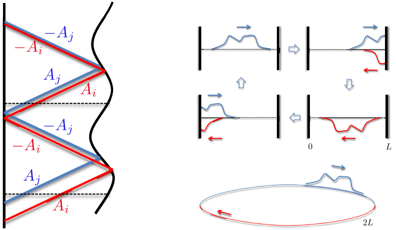
# Motivation: Floquet systems

## Time crystals

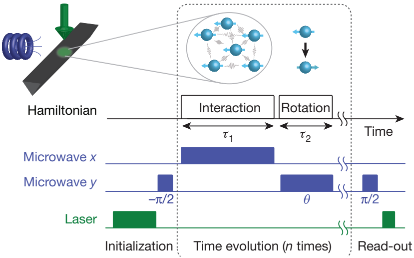



Zhang et al., Nature (2017)

## EM field in modulated cavity



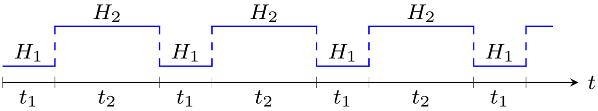

Martin, Ann. Phys. (2019)




Choi et al., Nature (2017)

# Driven inhomogeneous 1+1D CFT

Floquet setup: 2-step drive




with $H_1$, $H_2$ inhomogeneous CFT Hamiltonians with $v_1(x) \neq v_2(x)$.

Floquet operator:

$$U_F = \mathrm{e}^{-\mathrm{i}H_1 t_1} \mathrm{e}^{-\mathrm{i}H_2 t_2}$$

with $t_{\text{cyc}} = |t_1| + |t_2|$.

# Outline

- Inhomogeneous conformal field theory
- Main tools
- Floquet time evolution
- Phase diagrams and flow of energy and excitations
- Entanglement entropy

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# Inhomogeneous 1+1D CFT

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In Fourier space: Two commuting copies of the Virasoro algebra.

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# Example: Inhomogeneous Luttinger model

Inhomogeneous CFT with $c = 1$ given by

$$H = \int_{-L/2}^{L/2} \mathrm{d}x\, v_F(x) \Big[ {:}\psi_+^\dagger(x)\,(-\mathrm{i}\partial_x)\,\psi_+(x){:} + {:}\psi_-^\dagger(x)\,(+\mathrm{i}\partial_x)\,\psi_-(x){:} \Big]$$
$$+ \lambda\pi \int_{-L/2}^{L/2} \mathrm{d}x\, v_F(x) \big[\rho_+(x) + \rho_-(x)\big]\big[\rho_+(x) + \rho_-(x)\big] - L\mathcal{E}_0$$

with $\rho_\pm(x) = {:}\psi_\pm^\dagger(x)\psi_\pm(x){:}$ and fermionic fields $\psi_\pm^{(\dagger)}(x)$ satisfying

$$\big\{\psi_r(x), \psi_{r'}^\dagger(y)\big\} = \delta_{r,r'}\delta(x-y), \qquad \big\{\psi_r(x), \psi_{r'}(y)\big\} = 0.$$

After bosonization:

$$T_\pm(x) = \pi {:}\left[ \frac{1+K}{2\sqrt{K}} \rho_\pm(x) + \frac{1-K}{2\sqrt{K}} \rho_\mp(x) \right]^2{:} - \frac{\pi}{12L^2}$$

with $v(x) = v_F(x)\sqrt{1+2\lambda}$ and $K = 1/\sqrt{1+2\lambda}$. (Require $\lambda > -1/2$)

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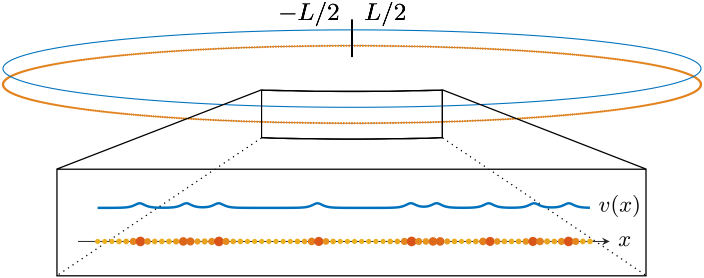
# Example: Effective description of inhomogeneous spin chains




P.M., arXiv:1912.04821, accepted in Ann. Henri Poincaré

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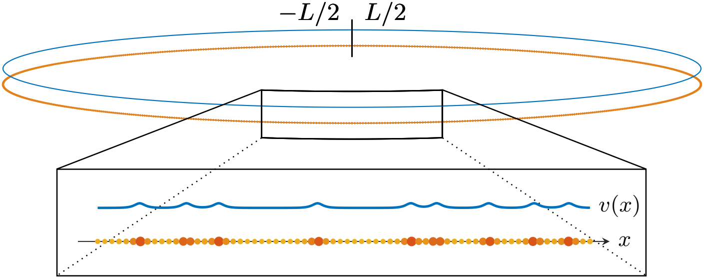


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For concreteness, let

$$J_j = \frac{v_F(x_j) + v_F(x_{j+1})}{2a \sin(ak_F)} > 0$$

with $v_F(x)$ a smooth function and $ak_F$ close to $\pi/2$. Then as effective description we obtain an inhomogeneous local Luttinger model with

$$v(x) = v_F(x)\sqrt{1 + 4\Delta \sin(ak_F)/\pi}, \quad K = \frac{1}{\sqrt{1 + 4\Delta \sin(ak_F)/\pi}}.$$

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# Diffeomorphism representations

## Orientation-preserving diffeomorphisms

$$f(x) = \int_0^x \mathrm{d}x' \, \frac{v_0}{v(x')}, \qquad \frac{1}{v_0} = \frac{1}{L} \int_{-L/2}^{L/2} \frac{\mathrm{d}x'}{v(x')}$$

$\implies f(x+L) = f(x) + L$ and $f'(x) > 0 \implies f \in \widetilde{\mathrm{Diff}}_+(S^1)$.

Gawędzki, Langmann, P.M., J. Stat. Phys. (2018)
P.M., arXiv:1912.04821, accepted in Ann. Henri Poincaré

Projective unitary representations given by

$$U_\pm(f) = I \mp \mathrm{i}\varepsilon \int_{-L/2}^{L/2} \mathrm{d}x \, \zeta(x) T_\pm(x) + o(\varepsilon)$$

for infinitesimal $f(x) = x + \varepsilon\zeta(x)$. Adjoint action:

$$U_\pm(f) T_\pm(x) U_\pm(f)^{-1} = f'(x)^2 T_\pm(f(x)) - \frac{c}{24\pi}\{f(x), x\},$$
$$U_\pm(f) T_\mp(x) U_\pm(f)^{-1} = T_\mp(x).$$

Goodman, Wallach, J. Func. Anal. (1985)

# Virasoro-Bott group

Bott cocycle:

$$U_\pm(f_1)U_\pm(f_2) = \mathrm{e}^{\pm \mathrm{i} c B(f_1,f_2)/24\pi} U_\pm(f_1 \circ f_2),$$

$$B(f_1,f_2) = \frac{1}{2}\int_{-L/2}^{L/2} \mathrm{d}x \, [\log f_2'(x)]' \, \log[f_1'(f_2(x))].$$

Virasoro-Bott group: Central extension of $\widetilde{\mathrm{Diff}}_+(S^1)$ by $B(f_1,f_2)$.

Associated Lie algebra: Virasoro algebra. Two commuting copies:

$$\left[L_n^\pm, L_m^\pm\right] = (n-m)L_{n+m}^\pm + \frac{c}{12}(n^3-n)\delta_{n+m,0}, \qquad \left[L_n^\pm, L_m^\mp\right] = 0,$$

where

$$T_\pm(x) = \frac{2\pi}{L^2}\sum_{n=-\infty}^{\infty} \mathrm{e}^{\pm\frac{2\pi \mathrm{i} n x}{L}} \left(L_n^\pm - \frac{c}{24}\delta_{n,0}\right).$$

E.g.: Khesin, Wendt, *The Geometry of Infinite-Dimensional Groups* (2009)

# Time evolution of operators

For local observables

$$\mathcal{O}(x;t) = \mathrm{e}^{\mathrm{i}Ht}\mathcal{O}(x)\mathrm{e}^{-\mathrm{i}Ht}.$$

Generalized light-cone coordinates $x_t^{\mp} = x_t^{\mp}(x)$ given by

$$x_t^{\mp}(x) = f^{-1}(f(x) \mp v_0 t)$$

using our $f \in \widetilde{\mathrm{Diff}}_+(S^1)$. Obtained by inserting $U_\pm(f)^{-1}U_\pm(f)$ above.

For Virasoro primary fields and the energy-momentum tensor:

$$\Phi(x;t) = \left[\frac{\partial x_t^-}{\partial x}\right]^{\Delta_\Phi^+}\left[\frac{\partial x_t^+}{\partial x}\right]^{\Delta_\Phi^-}\Phi(x_t^-, x_t^+),$$

$$T_\pm(x;t) = \left[\frac{\partial x_t^\mp}{\partial x}\right]^2 T_\pm(x_t^\mp) - \frac{c}{24\pi}\{x_t^\mp, x\}.$$

# Complementary approach

Inhomogeneous Tomonaga-Luttinger liquids

$$H_{\mathrm{iTLL}} = \frac{1}{2}\int_{-L/2}^{L/2} \mathrm{d}x \left( \frac{v(x)}{K(x)} \pi_\phi(x)^2 + v(x)K(x)[\partial_x \phi(x)]^2 \right)$$

with $[\phi(x), \pi_\phi(x')] = \mathrm{i}\delta(x - x')$. Corresponds to Lagrangian density

$$\mathcal{L} = \frac{v}{2}\sqrt{-h}K(x)h^{\mu\nu}(\partial_\mu \phi)(\partial_\nu \phi)$$

in curved spacetime

$$\mathrm{d}s^2 = v(x)^2 \mathrm{d}t^2 - \mathrm{d}x^2 = h_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu \qquad (x^0 = vt,\ x^1 = x).$$

Using Euclidean CFT:

Dubail, Stéphan, Viti, Calabrese, SciPost Phys. (2017)
Dubail, Stéphan, Calabrese, SciPost Phys. (2017)
Ruggiero, Brun, Dubail, SciPost Phys. (2019)

For $v(x)/K(x) = $ constant:

Gluza, P.M., Sotiriadis, arXiv:2104.07751

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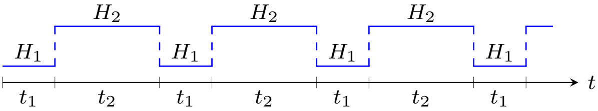
# Driven inhomogeneous 1+1D CFT

Floquet setup: 2-step drive




with $H_1$, $H_2$ inhomogeneous CFT Hamiltonians with $v_1(x) \neq v_2(x)$.

Floquet operator:

$$U_F = \mathrm{e}^{-\mathrm{i}H_1 t_1} \mathrm{e}^{-\mathrm{i}H_2 t_2}$$

with $t_{\mathrm{cyc}} = |t_1| + |t_2|$.

# Special case

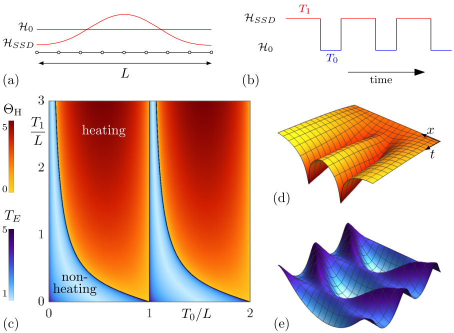
Sine-square-deformed (SSD) CFT:

$$H_{\text{SSD}} = \frac{2\pi v}{L}\left(L_0^+ - \frac{L_1^+ + L_{-1}^+}{2} + L_0^- - \frac{L_1^- + L_{-1}^-}{2}\right).$$

Wen, Wu, arXiv:1805.00031
Fan, Gu, Vishwanath, Wen, Phys. Rev. X (2020)
Lapierre, Choo, Tauber, Tiwari, Neupert, Chitra, Phys. Rev. Research (2020)

Homogeneous CFT: $H_{\text{homog.}} = \frac{2\pi v}{L}\left(L_0^+ + L_0^-\right).$

Phase diagram:




Möbius transformations:

$$\tilde{z}_1 = \frac{az+b}{cz+d}, \quad a = a(T_0, T_1), \text{ etc.}$$

$$\tilde{z}_n = \frac{(\gamma_1 - \eta^n \gamma_2)z + (\eta^n - 1)\gamma_1\gamma_2}{(1-\eta^n)z + \eta^n\gamma_1 - \gamma_2}$$

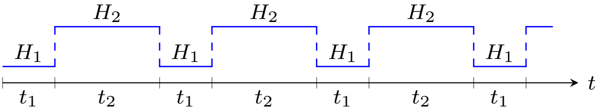
with

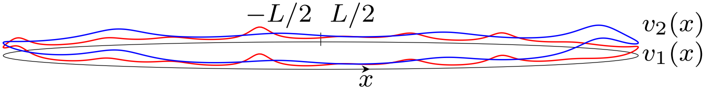
$$\begin{cases} \gamma_1 &= \gamma_1(a,b,c,d) \\ \gamma_2 &= \gamma_2(a,b,c,d) \\ \eta &= \eta(a,b,c,d) \end{cases}.$$

Hamiltonians

$$H_j = \int_{-L/2}^{L/2} \mathrm{d}x\, v_j(x)\big[T_+(x) + T_-(x)\big] \qquad (j = 1, 2)$$

with smooth $L$-periodic functions $v_1(x) > 0$ and $v_2(x) > 0$.




Lapierre, P.M., Phys. Rev. B (2021)

Special case of SSD CFT:

$$v_{\mathrm{SSD}}(x) = 2v\cos^2(\pi x/L).$$

# Geometric approach

Encode $U_F$ into circle diffeomorphisms $f_{\pm}(x)$ and study those. Yields a correspondence with classical dynamical systems on the circle.

Lapierre, P.M., Phys. Rev. B (2021)

# 2-step Floquet drive

For local observables

$$\mathcal{O}(x;t) = U_F^{-n}\mathcal{O}(x)U_F^n, \qquad U_F = \mathrm{e}^{-\mathrm{i}H_1t_1}\mathrm{e}^{-\mathrm{i}H_2t_2}, \qquad t = nt_{\mathrm{cyc}}.$$

Orientation-preserving diffeomorphisms

$$f_j(x) = \int_0^x \mathrm{d}x'\,\frac{v_{j,0}}{v_j(x')}, \qquad \frac{1}{v_{j,0}} = \frac{1}{L}\int_{-L/2}^{L/2}\frac{\mathrm{d}x}{v_j(x)}$$

$\implies f_j \in \widetilde{\mathrm{Diff}}_+(S^1)$. Insert $U_\pm(f_j)^{-1}U_\pm(f_j)$ around $\mathrm{e}^{\mp\mathrm{i}H_jt_j}$.

Consequence: Generalized light-cone coordinates $x_t^\mp(x)$ given by

$$x_{t+t_{\mathrm{cyc}}}^\mp(x) = f_\pm(x_t^\mp(x)), \qquad x_0^\mp(x) = x,$$

$$f_\pm(x) = f_2^{-1}\big[f_2\big(f_1^{-1}[f_1(x) \mp v_{1,0}t_1]\big) \mp v_{2,0}t_2\big].$$

# Time evolution of operators

Primary fields

$$\Phi(x;t) = \left[\frac{\partial x_t^-(x)}{\partial x}\right]^{\Delta^+} \left[\frac{\partial x_t^+(x)}{\partial x}\right]^{\Delta^-} \Phi(x_t^-(x), x_t^+(x)).$$

Components of the energy-momentum tensor

$$T_\pm(x;t) = \left[\frac{\partial x_t^\mp(x)}{\partial x}\right]^2 T_\pm(x_t^\mp(x)) - \frac{c}{24\pi}\{x_t^\mp(x), x\}.$$

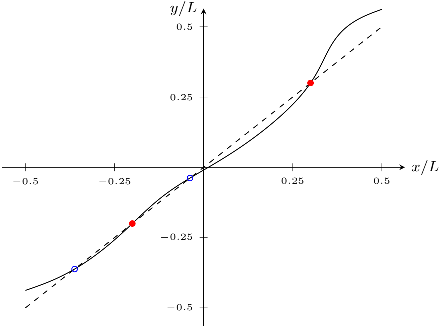
Here $\partial x_t^\mp(x)/\partial x = \prod_{m=0}^{n-1} f'_\pm(x_{mt_{\mathrm{cyc}}}^\mp(x))$ for $n > 0$.

Fixed points: Look for solutions $x_*^{\mp}$ to

$$x_*^{\mp} = f_{\pm}(x_*^{\mp}).$$

Tangent points: Critical values $x_c^{\mp} = x_*^{\mp}$ that additionally satisfy

$$1 = f'_{\pm}(x_c^{\mp}).$$




Unstable $f'_{\pm}(x_*^{\mp}) > 1$

Stable $f'_{\pm}(x_*^{\mp}) < 1$

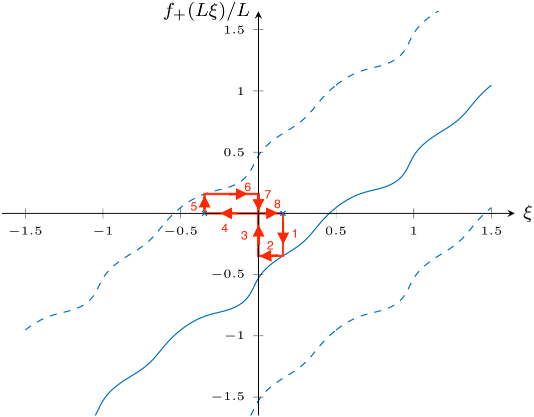
# More generally: Periodic points

Periodic points of period $p \in \mathbb{Z}^+$:

$$x^{\mp}_{*p} = f^p_{\pm}(x^{\mp}_{*p}), \qquad f^p_{\pm} = \underbrace{f_{\pm} \circ \ldots \circ f_{\pm}}_{p \text{ times}}.$$

Unstable: $f^{p\,\prime}_{\pm}(x^{\mp}_{*p}) > 1$. Stable: $f^{p\,\prime}_{\pm}(x^{\mp}_{*p}) < 1$. Critical: $f^{p\,\prime}_{\pm}(x^{\mp}_{\mathrm{c}p}) = 1$.

Example of periodic points with period 2:



# Heating and non-heating phases

Suppose that $x$ is a periodic point $x_{*p}^{\mp}$, then

$$\frac{\partial x_t^{\mp}(x)}{\partial x} = f_{\pm}^{p\,\prime}(x_{*p}^{\mp})^{n/p}, \qquad t = n t_{\mathrm{cyc}},\ n/p \in \mathbb{Z}^+.$$

If $f_{\pm}^{p\,\prime}(x_{*p}^{\mp}) > (<) 1$, this diverges (vanishes) exponentially as $t \to \infty$.

## Energy density

$$\mathcal{E}_1(x;t) = v_1(x)\big[T_+(x;t) + T_-(x;t)\big]$$

grows (decays) exponentially at unstable (stable) fixed points.

## Heating rate

$$\nu = \max_{p\in\mathbb{Z}^+,\, r=\pm,\, i\in\{1,\ldots,2N_p\}} \frac{2}{p t_{\mathrm{cyc}}} \log\big[f_r^{p\,\prime}(x_{*p,i}^{-r})\big].$$

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# Example 1: $g$SSD CFT

Special case: $v_1(x) = v_1$.

$g$SSD CFT: $v_2(x) = v_2 w(x/L)$ with

$$w(\xi) = 1 + g[2\cos^2(\pi\xi) - 1], \qquad g \in [0, 1).$$

Cf.: MacCormack, Liu, Nozaki, Ryu, J. Phys. A: Math. Theor. (2019)

Limiting cases: Homogeneous CFT: $g = 0$ SSD CFT: $g \to 1^-$

Dimensionless variables: $\xi = x/L$, $\tau_1 = v_1 t_1/L$, $\tau_2 = v_2 t_2/L$.

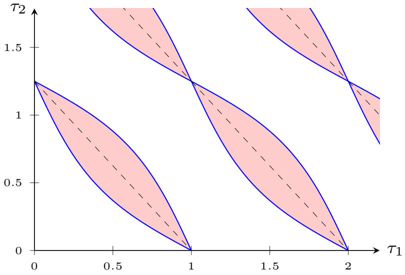
# Example 1: Phase diagram for $g$SSD CFT

Phase transition lines:

$$\tau_2 = \frac{2\arctan\left(\sqrt{(1-g)/(1+g)}\tan(\pi[1-\tau_1]/2)\right)}{\pi\sqrt{1-g^2}},$$

$$\tau_2 = \frac{1}{\sqrt{1-g^2}} - \frac{2\arctan\left(\sqrt{(1-g)/(1+g)}\tan(\pi\tau_1/2)\right)}{\pi\sqrt{1-g^2}}.$$

Phase diagram:




$g$SSD CFT for $g = 0.6$

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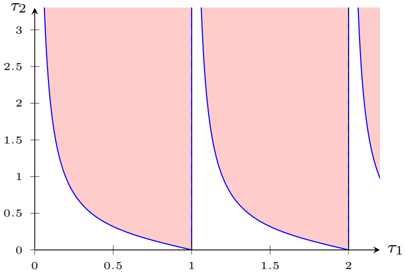



SSD CFT ($g \to 1^-$)

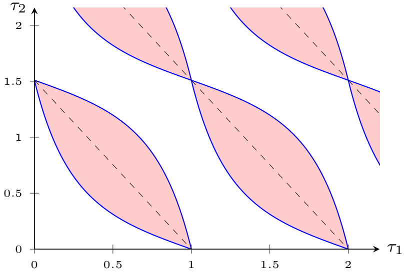
# Example 2: Gaussian-deformed CFT

As before but with

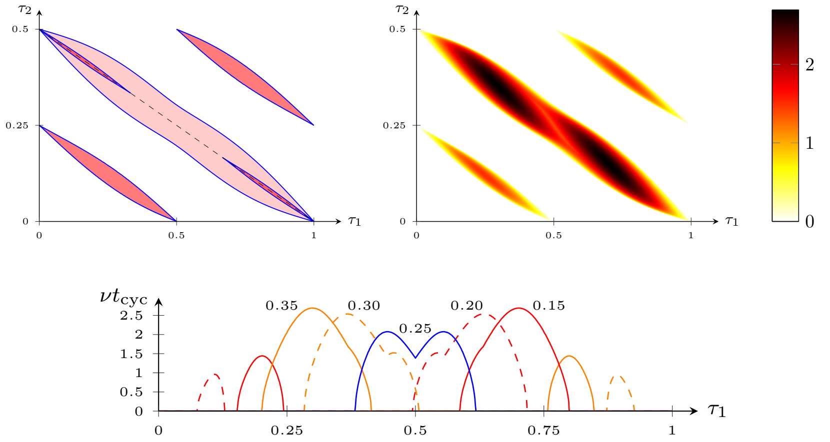
$$w(\xi) = A \exp\big(-(\xi/d)^2\big), \qquad A, d \in \mathbb{R}^+.$$

Explicit formulas with erfi instead of arctan.

Phase diagram:

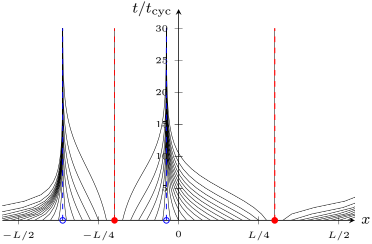



Gaussian-deformed CFT for $A = 2$, $d = 0.32$

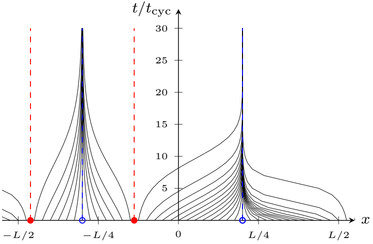



For $a = 6$, $b = 3$, and $k = 2$.

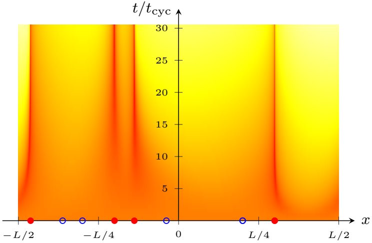
# Example 3: Fixed points at $(\tau_1, \tau_2) = (0.10, 0.45)$



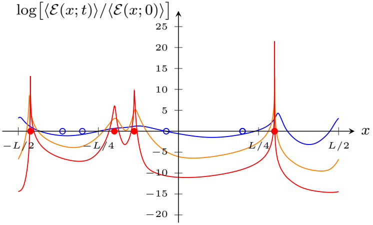

$x_t^-(x)$-trajectories




$x_t^+(x)$-trajectories

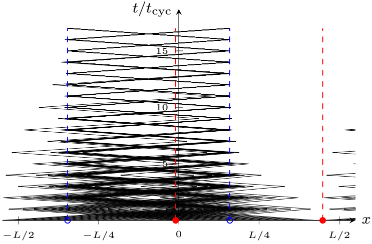



$\log\left(\sum_{r=\pm} |\langle 0|\Phi_r^\dagger(x,t)\Phi_r(x_0,0)|0\rangle|^2\right)$

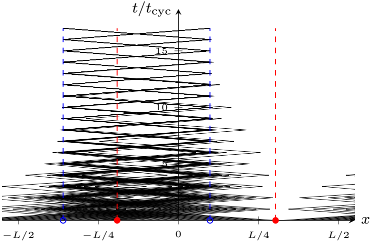



Energy density

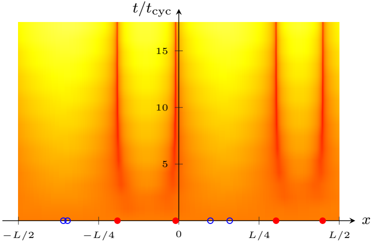
# Example 3: 2-periodic points at $(\tau_1, \tau_2) = (0.35, 0.08)$



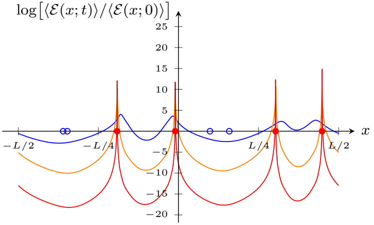

$x_t^-(x)$-trajectories




$x_t^+(x)$-trajectories




$\log\left(\sum_{r=\pm} |\langle 0|\Phi_r^\dagger(x,t)\Phi_r(x_0,0)|0\rangle|^2\right)$




Energy density

# Outline

- Inhomogeneous conformal field theory
- Main tools
- Floquet time evolution
- Phase diagrams and flow of energy and excitations
- Entanglement entropy

# Entanglement entropy: Computation

Entanglement entropy of subsystem on $A = [x, y]$ with the rest:

$$S_A(t) = \lim_{m \to 1} \frac{1}{1-m} \log\Big( \mathrm{Tr}\big[\hat{\rho}_A(t)^m\big]\Big)$$

with $\hat{\rho}_A(t) = U_F^n \hat{\rho}_A U_F^{-n}$, $t = nt_{\mathrm{cyc}}$, and $\hat{\rho}_A$ the reduced density matrix.

Using twist fields $\Phi_m(x;t)$:

$$S_A(t) = \lim_{m \to 1} \frac{1}{1-m} \log\big[\langle 0|\Phi_m(x;t)\Phi_m(y;t)|0\rangle\big].$$

Conformal weights $\Delta_m^{\pm} = (c/24)(m - 1/m)$.

Cardy, Castro-Alvaredo, Doyon, J. Stat. Phys. (2008)
Calabrese, Cardy, J. Stat. Mech. (2016)

Rigorous results for entanglement entropy in quantum field theory.

Longo, Xu, Adv. Math. (2018); Commun. Math. Phys. (2021)

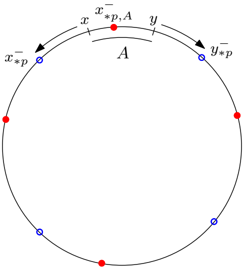
# Entanglement entropy: Results

Letting $x_t^{\mp} = x_t^{\mp}(x)$ and $y_t^{\mp} = x_t^{\mp}(y)$:

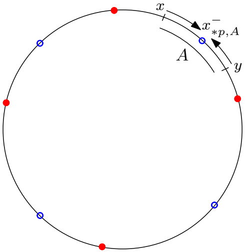
$$S_A(t) = \frac{c}{12}\big[S_+(t) + S_-(t)\big],$$

$$S_\pm(t) = -\log\left[\frac{\partial x_t^{\mp}}{\partial x}\frac{\partial y_t^{\mp}}{\partial y}\left(\frac{\pm \mathrm{i}\pi}{L\sin\!\big(\frac{\pi}{L}[x_t^{\mp} - y_t^{\mp} \pm \mathrm{i}0^+]\big)}\right)^2\right].$$

Two cases:



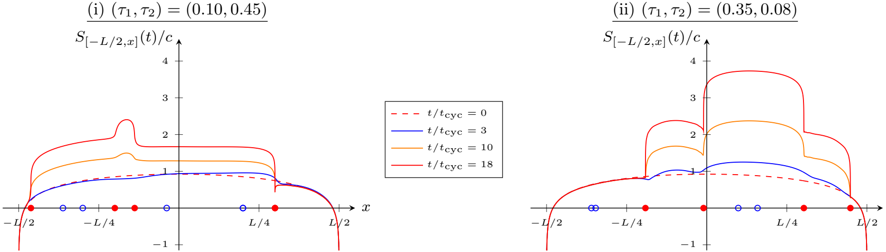

$S_+(t) \sim t$ for large $t$




$S_+(t) \sim$ constant for large $t$

If $t = 0$, then $S_A(0) = (c/3)\log\big[(L/\pi)\sin\big(\pi\ell/L\big)\big]$ for $\ell = x - y > 0$.

In Example 3:




Mutual information $I_{A;B}(t) = S_A(t) + S_B(t) - S_{A\cup B}(t)$:
Only neighboring unstable periodic points share entanglement that grows linearly at late times $\implies$ entanglement entropy is "bipartite".[1]

[1] If the number of unstable points for each component is even.

# Summary

- Exact analytical results for general inhomogeneous conformal field theory using projective unitary representations of $\widetilde{\mathrm{Diff}}_+(S^1)$.

- Geometric approach to inhomogeneous Floquet systems.

- Construct phase diagrams with heating/non-heating phases determined by presence/absence of periodic points.

- Energy and excitations accumulate exponentially fast at unstable periodic points.

- Kinks in entanglement entropy at unstable periodic points.

- Only neighboring unstable periodic points share linearly growing entanglement entropy at late times.

- Geometric approach is straightforward to apply to multi-step, random, chaotic, and quasi-periodic drives.

Thank you for your attention!