

Dagens teman

- Laplacetransformen (ZC, kap 7)
Egenskaper och räkneexempel

Laplaceformen

$$F(s) = \int_0^{\infty} f(t) e^{-st} dt.$$

(Laplace-)faltning

$$f(t) * g(t) = \int_0^t f(t - \tau) g(\tau) d\tau, t > 0.$$

Egenskaper hos laplacetransformen

| Funktion | Transform |
|--|--|
| $f(t)$ | $F(s)$ |
| $af(t) + bg(t)$ | $aF(s) + bG(s)$ |
| $e^{at}f(t)$ | $F(s - a)$ |
| $f(t - a) u(t - a), a > 0$ | $e^{-as} F(s)$ |
| $f(at), a > 0$ | $\frac{1}{a} F\left(\frac{s}{a}\right)$ |
| $\frac{1}{a} f\left(\frac{t}{a}\right), a > 0$ | $F(as)$ |
| $(f * g)(t)$ | $F(s) \cdot G(s)$ |
| $\frac{d}{dt} f(t)$ | $s F(s) - f(0)$ |
| $t f(t)$ | $-\frac{d}{ds} F(s)$ |
| $\frac{d^n}{dt^n} f(t)$ | $s^n F(s) - s^{n-1} f(0) -$ $- s^{n-2} f'(0) - \dots$ $\dots - s f^{(n-2)}(0) -$ $- f^{(n-1)}(0)$ |
| $t^n f(t)$ | $(-1)^n \frac{d^n}{ds^n} F(s)$ |
| $\int_0^t f(\tau) d\tau, t > 0$ | $\frac{F(s)}{s}$ |

Speciella transformer

| Funktion | Transform |
|-------------------|-----------------------|
| (t) | 1 |
| 1 | $\frac{1}{s}$ |
| t | $\frac{1}{s^2}$ |
| t^n | $\frac{n!}{s^{n+1}}$ |
| $(t - a), a > 0$ | e^{-as} |
| $u(t - a), a > 0$ | $\frac{e^{-as}}{s}$ |
| e^{at} | $\frac{1}{s - a}$ |
| e^{jat} | $\frac{1}{s - ja}$ |
| $\cos (at)$ | $\frac{s}{s^2 + a^2}$ |
| $\sin (at)$ | $\frac{a}{s^2 + a^2}$ |