Dagens teman

- Integraler av harmoniska funktioner (FM §7.1)
- Faltning (§7.2)
- Fouriertransformen (§7.3)

Sinus cardinalis:

- $$\operatorname{sinc} f = \begin{cases} \dfrac{\sin \pi f}{\pi f}, & \text{om } f \neq 0, \\ 1, & \text{om } f = 0. \end{cases}$$

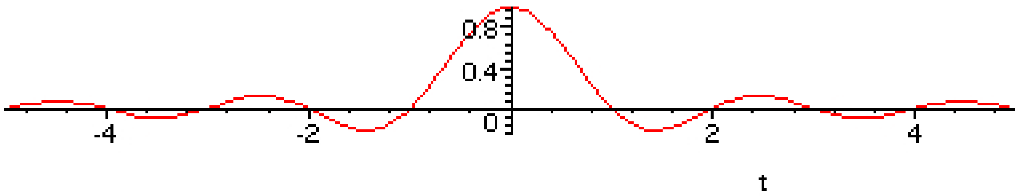
- $$\int_{-P/2}^{P/2} e^{2\pi i f t}\, df = \frac{\sin P\pi f}{\pi f} = P \operatorname{sinc} Pf,$$

- $$\int_{-P/2}^{P/2} e^{i\omega t}\, d\omega = \frac{\sin P\omega/2}{\omega/2} = P \operatorname{sinc} \left(P \frac{\omega}{2\pi} \right).$$

δ-*pulsen som summa av alla harmoniska signaler*:

- $$\int_{-\infty}^{\infty} e^{2\pi i f t}\, df = \delta(t),$$

- $$\int_{-\infty}^{\infty} e^{i\omega t}\, d\omega = 2\pi\, \delta(t),$$



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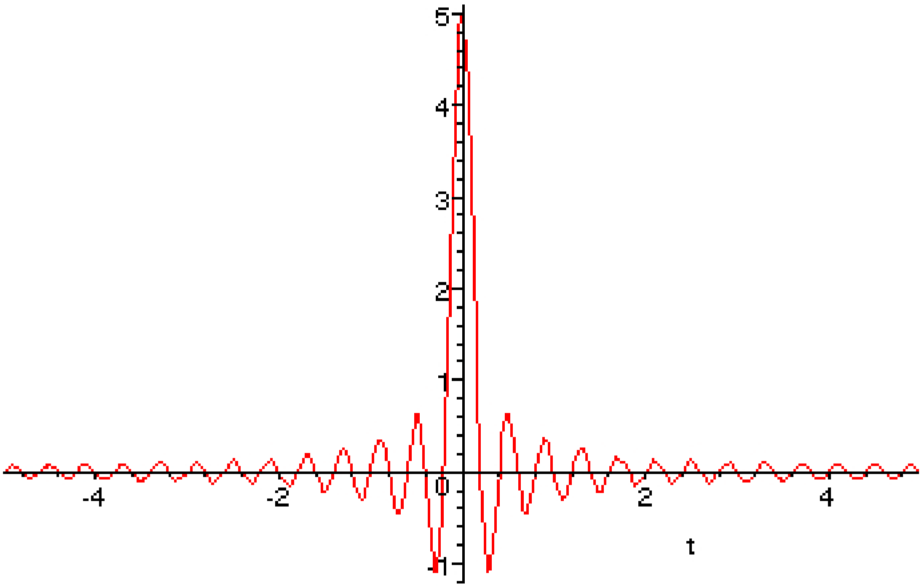

5 sinc $5t$

Fouriertransformen

Syntesekvationen:

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)\, e^{j\omega t} d\omega$$

Analysekvationen:

$$X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t} dt$$

Parsevals formel:

$$\int_{-\infty}^{\infty} |x(t)|^2 dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(\omega)|^2 d\omega$$

Egenskaper hos faltning

$$(x*y)(t) = \int_{-\infty}^{\infty} x(\tau)\, y(t-\tau)\, d\tau.$$

$$x * y = y * x.$$

$$x * \delta(t) = x.$$

$$x * \delta'(t) = x'(t), \quad x * \delta^{(p)}(t) = x^{(p)}(t)$$

$$x * (a_1 y_1 + a_2 y_2) = a_1(x * y_1) + a_2(x * y_2)$$

$$x * \sum_n a_n y_n = \sum_n a_n (x * y_n).$$

$$x(t) * \int_{-\infty}^{\infty} y(t,f)\, df = \int_{-\infty}^{\infty} x(t) * y(t,f)\, df$$

$$(x*y)*z = x*(y*z).$$

Egenskaper hos fouriertransformen

Funktion	Transform
Om $x(t)$	$Z(\omega)$
så $Z(t)$	$2\pi \cdot x(-\omega)$
$x(t)$	$X(\omega)$
$e^{i\omega_0 t} x(t)$	$X(\omega - \omega_0)$
$x(t - t_0)$	$e^{-i\omega t_0} X(\omega)$
$x(at),\ a \neq 0$	$\frac{1}{\lvert a \rvert} X\left(\frac{\omega}{a}\right)$
$x(-t)$	$X(-\omega)$
$(x * y)(t)$	$X(\omega) \cdot Y(\omega)$
$x(t) \cdot y(t)$	$\frac{1}{2\pi}(X * Y)(\omega)$
$\frac{d}{dt} x(t)$	$i\omega X(\omega)$
$t\, x(t)$	$i \frac{d}{d\omega} X(\omega)$
$\left(\frac{d}{dt}\right)^n x(t)$	$(i\omega)^n X(\omega)$
$t^n x(t)$	$i^n \frac{d^n}{d\omega^n} X(\omega)$
Sampling av $x(t)$ med sampelavstånd T	$2\pi/T$-periodisk fortsättning av $1/T \cdot X(\omega)$
L-periodisk fortsättning av $x(t)$	Sampling av $2\pi/L \cdot X(\omega)$ med avstånd $2\pi/L$

Speciella transformer

Funktion	Transform
$\delta(t)$	1
1	$2\pi\,\delta(\omega)$
$\delta(t - t_0)$	$e^{-i\omega t_0}$
$e^{i\omega_0 t}$	$2\pi\,\delta(\omega - \omega_0)$
$\delta(t - t_0) + \delta(t + t_0)$	$e^{-i\omega t_0} + e^{i\omega t_0} = 2\cos(\omega t_0)$
$\cos(\omega_0 t)$	$\pi\,(\delta(\omega - \omega_0) + \delta(\omega + \omega_0))$
$\delta(t + t_0) - \delta(t - t_0)$	$e^{i\omega t_0} - e^{-i\omega t_0} = 2i\sin(\omega t_0)$
$\sin(\omega_0 t)$	$-\pi i\,(\delta(\omega - \omega_0) - \delta(\omega + \omega_0))$
$\sum_{n=-\infty}^{\infty} \delta(t - nT)$	$2\pi/T \sum_{n=-\infty}^{\infty} \delta(\omega - 2\pi n/T)$
$u(t)$	$\frac{1}{i\omega} + \pi\,\delta(\omega)$
$\text{sign}(t)$	$\frac{2}{i\omega}$
$\text{rect}(t/P)$	$P\,\text{sinc}(P\omega/(2\pi))$
$\text{sinc}(t/(2\pi))$	$2\pi\,\text{rect}(\omega)$
$\text{sinc}(t)$	$\text{rect}(\omega/(2\pi))$