LUNDS UNIVERSITET

Matematikcentrum
Matematik MNF

Tentamensskrivning
Analytiska funktioner
Onsdagen 31 oktober 2001, 8.00–13.00

Inga hjälpmedel. Använd institutionens papper och skriv på bara den ena sidan. Fyll i omslaget fullständigt och skriv initialer på varje ark. Skriv tydligt. Ge klara och kortfattade motiveringar; rita gärna figur i förekommande fall.

1. Find all (complex) roots of the equation $\sin z = 2$.

2. Let D be an open set in the plane, and suppose that f and g are holomorphic there; you can assume that this means that each function is continuously differentiable and satisfies the Cauchy-Riemann equations on D. Show that the product fg also is holomorphic in D by verifying the Cauchy-Riemann equations for it.

3. Do a Laurent series expansion of the function

$$f(z) = e^{2z} + e^{1/z}$$

around the origin. Also, calculate the integral

$$\int_\Gamma z^{100}\, f(z)\, dz,$$

where Γ is the curve $|z| = 2$ in the counter-clockwise direction.

4. Evaluate the integral

$$\int_{-\infty}^{+\infty} \frac{\cos x}{1+x^4}\, dx.$$

5. Determine whether there exist functions f_1, f_2, f_3, with the following properties:
(a) f_1 is holomorphic in the unit disk $|z| < 1$ with $f_1(1/n) = e^{-1/n}$ for all $n = 2, 3, 4, 5, \ldots$,
(b) f_2 is holomorphic in the unit disk $|z| < 1$ and has $\operatorname{Re}[f_2(z)^2] = 2$ there, and
(c) f_3 is holomorphic in the whole plane with real part $x^3 - 3xy^2$, where $z = x + iy$ is the complex variable.

6. Suppose we are given that F is an entire function, and that

$$|F(z)| \le K\left(1 + |z|^{5/2}\right)$$

holds for all complex z, for some positive constant K. What can you say about F?