SEMINARIUM I ANALYS OCH DYNAMISKA SYSTEM

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Bounds for the best constant for the weak type $(1,1)$ estimate for the centered maximal function.

Abstract Let $x = (x_1, \ldots, x_d)$ denote a point in $\mathbf{R}^d$ and let the cube $B(x,r)$ be defined by $B(x,r) = \{y; |y_i - x_i| < r, 1 \le i \le d\}$. Let $|\cdot|$ denote the Lebesgue measure in $\mathbf{R}^d$. The centered maximal function $Mf(x)$ of a function on $\mathbf{R}$ is defined by

$$Mf(x) = \sup_{r>0} \frac{1}{|B(x,r)|} \int_{B(x,r)} |f(y)| dy$$

The maximal function satisfies a so called weak type (1,1) estimate.

$$|\{x; Mf(x) > \lambda\} \le C\|f\|_{L_1}$$

Here the constant C may depend on the dimension d Let A_d be the best possible constant C in this weak type estimate. In 1983 we (E.M. Stein & S.) showed that $A_d = O(d \log d)$. It was recently shown by J.M.Aldaz that $A_d \to \infty$ as $d \to \infty$. On the seminar I will show Aldaz construction, with some change in the estimates. This will give a lower estimate of how fast the best constant A_d goes to infinity with d.