VETENSKAPSRÅDET
THE SWEDISH RESEARCH COUNCIL

| Kansliets noteringar Kod [redacted] | Dnr |
|---|---|

# 2006
# Junior research position

Area of science
Natural and Engineering Sciences
Announced grants
Research Grants NT 25 April 2006
Total amount for which applied (kSEK)

| 2007 | 2008 | 2009 | 2010 | 2011 |
|---|---|---|---|---|
| | | | | |

## APPLICANT


Name(Surname, First name)
Jonsson, Jakob
Email address
[redacted]
Phone
[redacted]

Date of birth
720914-[redacted]
Academic title
Phd
Date of doctoral exam
2005-06-16

Sex
Male
Position
Post-doctoral grant


## WORKING ADDRESS


University/corresponding, Department, Section/Unit, Address, etc.
Technische Universität Berlin
Institut für Mathematik
Diskrete Geometrie
Straße des 17. Juni 136
10623 Berlin, Germany


## PROPOSED HOST ORGANISATION

University/corresponding, Department, Section/Unit, Address, etc.
Kungl Tekniska högskolan
Institutionen för matematik
Avdelningen för matematik
Lindstedtsvägen 25
10044 Stockholm, Sweden

## DESCRIPTIVE DATA

Project title, Swedish (max 200 char)
Samband mellan topologisk kombinatorik och andra områden inom matematiken

Project title, English (max 200 char)
Interactions between topological combinatorics and other fields of mathematics


Abstract (max 1500 char)
My main field of research has been topological combinatorics, the emphasis being on homology, homotopy type, and Euler characteristic of simplicial complexes. My wish is to pursue this research in new directions.

To start with, I want to study the theory of group actions on simplicial complexes, the ultimate goal being to apply such techniques to concrete problems. For example, the simplicial complex known as the matching complex has a well-understood rational homology, but only little is known about torsion in the integral homology. Representation theory proved extremely successful in the rational case and is probably indispensable also in the integral case.

A somewhat related goal is to get better acquainted with the whole area surrounding the evasiveness conjecture for vertex-transitive simplicial complexes. Again, representation theory is central.

Another goal is to examine connections between topological combinatorics and commutative algebra via Stanley-Reisner and similar theories. Such connections already appear briefly in my work, but I want to deepen my knowledge further.

**Finally, I want to learn more about the geometry of simplicial and cell complexes. An important problem is that of finding a geometric realization of a complex as the boundary complex of a convex polytope. Indeed, open questions of this kind appear in my research.**




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Kod [redacted]
Name of Applicant
Jonsson, Jakob
Date of birth
720914[redacted]

Abstract language
English
Keywords, Medline(M)
topological combinatorics, graph theory, simplicial complex, monotone graph property, group actions
Research areas
Mathematics & Engineering Mathematics
Review panel
NT-R
Classification codes (SCB) in order of priority
140105
Aspects

Application is also submitted to
similar to: identical to:

## HUMAN AND ANIMAL STUDIES

Human studies
No approved Human studies.
Animal studies
No approved Animal studies.

## ENCLOSED APPENDICES

A, B, C, S

## APPLIED FUNDING: THIS APPLICATION

Funding period (planed start and end date)
2007-07-01 -- 2011-06-30

| Scientific equipment < 170 kSEK, materials, other costs (kSEK) | 2007 | 2008 | 2009 | 2010 | 2011 |
|---|---|---|---|---|---|
| Total, other costs (kSEK): | | | | | |

| Scientific equipment between 170 kSEK and 2000 kSEK (kSEK) | 2007 | 2008 | 2009 | 2010 | 2011 |
|---|---|---|---|---|---|
| Total, equipment (kSEK): | | | | | |

Total amount for which applied (kSEK)

| 2007 | 2008 | 2009 | 2010 | 2011 |
|---|---|---|---|---|
| | | | | |

Funding of salaried positions and stipends are set to predetermined standard amounts.

## ALL FUNDING

Other VR-projects (granted and applied) by the applicant and co-workers, if applic. (kSEK)

Funds received by the applicant from other funding sources, incl ALF-grant (kSEK)

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THE SWEDISH RESEARCH COUNCIL

Kod
[redacted]

Name of Applicant
Jonsson, Jakob

Date of birth
720914[redacted]

## POPULAR SCIENCE DESCRIPTION

Popular science heading and description (max 4500 char)

Populärvetenskaplig beskrivning

Mitt ämne är topologisk kombinatorik, en gren inom matematiken som förenar å ena sidan grafteori och diskret matematik och å andra sidan algebraisk topologi.

Inom grafteorin studerar man objekt kallade "grafer". Sådana objekt består av ett antal punkter, kallade "hörn", och en uppsättning linjestycken, kallade "kanter", som sammanbinder olika par av hörn. Ett praktiskt exempel är en schematisk vägkarta där städer representeras av hörn och vägar mellan städerna representeras av kanter. Grafteorin är ett delområde av den diskreta matematiken, som lite slarvigt kan beskrivas som det område av matematiken där man studerar problem av "ändlig" karaktär. Som exempel kan nämnas enumerativa problem, vilka går ut på att bestämma antalet objekt i en ändlig mängd, exempelvis antalet möjliga lottorader med ett visst antal rätt. Andra diskreta problem handlar om att ordna element på ett sätt så att de uppfyller vissa villkor. Några klassiska exempel är latinska kvadrater och färgläggningar av politiska kartor.

Inom den algebraiska topologin, eller snarare den förenklade variant som jag är intresserad av, studerar man geometriska objekt: punkter, cirklar, månghörningar, kuber, klot och så vidare. Man är framför allt intresserad av att undersöka när ett objekt kan överföras i ett annat via deformeringar och omformningar. Exempelvis kan man "platta till" en tredimensionell kub så att man får en tvådimensionell kvadrat. Man kan även "krama om" kuben så att man får ett tredimensionellt klot. Genom att införa vissa regler för vad man får och inte får göra erhåller man en indelning av alla geometriska objekt i klasser av "ekvivalenta" objekt, där varje enskild klass innehåller alla objekt som kan omformas i varandra enligt de givna reglerna. En typisk regel är att förbjuda omformningar där man bryter sönder ett objekt. En annan regel är att inte tillåta att man lägger till eller fyller igen ett "hål" som är helt inneslutet i objektet. Exempelvis är det inte tillåtet att gröpa ur ett massivt klot så att man bara får kvar klotets hölje, det vill säga en tredimensionell sfär. En grundpoäng med algebraisk topologi är att försöka fånga kärnegenskaperna hos ett geometriskt objekt, till exempel hur många "hål" det innesluter.

Den brygga från diskret matematik till algebraisk topologi som utgör själva hjärtat i min forskning går ut på att man tolkar familjer av mängder som geometriska objekt, så kallade "simpliciella komplex". Dessa objekt är uppbyggda av byggklossar kallade "simplex", en för varje dimension. De noll- och endimensionella klossarna är desamma som för grafer, alltså hörn respektive kanter. I den närmast högre dimensionen hittar vi triangelskivor, alltså tvådimensionella objekt vars rand är uppbyggd av en triangel bestående av tre kanter och tre hörn. Nästa objekt är tetraederblocket, vars rand består av fyra triangelskivor, sex kanter och fyra hörn. Här kan man skönja ett mönster som sedan fortsätter i högre dimensioner; för att bilda ett simplex av dimension d behöver man d+1 simplex av dimension d-1, och resultatet har d+1 hörn.

Man erhåller kopplingen mellan mängder och simplex genom att identifiera varje element i en given mängd med ett hörn och sedan bilda ett simplex med dessa hörn. Exempelvis ger en mängd med två element upphov till ett simplex med två hörn, alltså en kant. En mängd med tre eller fyra element ger på samma sätt upphov till en triangelskiva eller ett tetraederblock. Om vi har en familj med mängder erhåller vi sålunda en hel uppsättning med simplex, närmare bestämt ett simplex för varje mängd. Vi konstruerar vårt simpliciella komplex genom att klistra ihop byggklossarna på det "naturliga" sättet; hörn som svarar mot samma element identifieras, liksom kanter, triangelskivor och simplex av högre dimension. Genom att studera det erhållna objektet ur ett topologiskt perspektiv kan man sedan dra intressanta slutsatser om den ursprungliga familjen.

Det beskrivna området i gränslandet mellan diskret matematik och topologi är ett mycket livaktigt forskningsområde, både i Sverige och i utlandet, kanske framför allt i USA, Tyskland, Israel och Ryssland. Det finns många exempel på hur ett svårt problem inom diskret matematik har funnit en lösning genom att man har tillämpat topologiska metoder, och det är bland annat detta jag har förhoppningar om att kunna uppnå med min forskning.



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Kod
[redacted]

Name of applicant
Jonsson, Jakob

Date of birth
720914[redacted]

Title of research programme
Interactions between topological combinatorics and other fields of mathematics

# Appendix A

## Research programme

# 1 Specific goals

My main field of research has been topological combinatorics, the emphasis being on homology, homotopy type, and Euler characteristic of simplicial complexes. My wish is to pursue this research in new directions:

(1) I want to study the theory of group actions on simplicial complexes, the ultimate goal being to apply such techniques to concrete problems. One example is the problem of computing the integral homology of the matching complex. I describe this problem in greater detail in the overview below.

(2) I aim to get better acquainted with the whole area surrounding the evasiveness conjecture for different classes of vertex-transitive simplicial complexes such as monotone graph properties.

(3) I want to examine connections between topological combinatorics and commutative algebra via Stanley-Reisner and similar theories. Such connections already appear briefly in my work (see below), but I want to deepen my knowledge further.

(4) I wish to learn more about the geometry of simplicial and cell complexes. An important problem is that of finding a geometric realization of a complex as the boundary complex of a convex polytope. Indeed, open questions of this kind appear in my research; see below.

# 2 Overview

In the previous section, I described four different directions for my future research. Here I provide some background for these four directions.

(1) My doctoral thesis [3] evolved around graph complexes, which are graph families on a fixed vertex set closed under deletion of edges. Equivalently, a graph complex $\Delta$ has the property that if $G \in \Delta$ and $e$ is an edge in $G$, then the graph obtained from $G$ by removing $e$ is also in $\Delta$. Since the vertex set is fixed, we may identify each graph in $\Delta$ with its edge set and hence interpret $\Delta$ as a simplicial complex. In particular, we may realize $\Delta$ as a geometric object and hence analyze its topology. If a graph complex



$\Delta$ is invariant under the action of the symmetric group on the underlying vertex set, then we say that $\Delta$ is a *monotone graph property*. In my thesis, I examined numerous monotone graph properties, some examples being the properties of being bipartite, $k$-connected, and non-Hamiltonian.

While several monotone graph properties in my thesis turn out to be easy to describe in terms of homology and homotopy type, there are many examples of complexes with a much more complicated structure, including torsion in the homology. This is where I believe that representation theory might be a useful tool.

Let me discuss the most well-studied monotone graph property in some detail. The *matching complex* $\mathsf{M}_n$ is the simplicial complex of matchings in the complete graph on $n$ vertices; a matching is a set of edges such that no two edges in the set have a vertex in common. Thanks to a beautiful result due to Bouc [1], the rational homology of $\mathsf{M}_n$ is well-understood, but only little is known about torsion in the integral homology. Building on Bouc's work, Shareshian and Wachs [9] provided a near-complete characterization of the bottom nonvanishing homology group of $\mathsf{M}_n$. Proceeding in the same spirit in recent work [4], I prove some new results about the existence and nonexistence of $p$-torsion in higher-degree homology groups of $\mathsf{M}_n$ for different primes $p$, most notably $p = 3$ and $p = 5$.

Yet, regarding the full picture, we all remain stumbling in the dark. To make substantial progress on the problem, it seems that we would need completely new ideas. Given that representation theory proved extremely successful in Bouc's analysis of the rational homology, it seems reasonable to assume that the same is true for the integral homology. The right approach might be to combine representation theory with appropriately defined exact sequences relating the homology of different matching complexes.

(2) Karp's longstanding evasiveness conjecture for monotone graph properties states that every monotone graph property is either a simplex or evasive; an evasive simplicial complex is a collapsible complex with some further structure. Using group theory, Kahn, Saks, and Sturtevant [6] settled the conjecture in the case that the size of the underlying vertex set is a prime power, but the general case remains open. Another reason for improving my knowledge on group actions and representation theory is to better understand this conjecture. Proving the conjecture in its full strength is likely to be excessively difficult; for what we know, the conjecture may well be false. Instead, one may focus on stronger versions of the conjecture, pushing the

limits for what is certainly *not* true. For example, in earlier research, I stumbled over a $\mathbb{Q}$-acyclic monotone graph property on six vertices, and one may ask whether there are similar examples of $\mathbb{Z}$-acyclic, contractible or even collapsible nontrivial monotone graph properties. Again, group theory might be of great help to get a better picture.

(3) Many deep results in topological combinatorics and commutative algebra rely on the Stanley-Reisner correspondence, which associates a square-free monomial ideal to any simplicial complex and vice versa. In my research, I have scratched the surface of this correspondence on a few occasions. In recent joint work with Volkmar Welker [5], we use this correspondence to examine a certain simplicial complex and a related polynomial ideal of Pfaffians, proving results about both objects. The objects reappeared in even more recent work of Krattenthaler [7] and Rubey [8]. Defining elegant bijections between different monomial ideals, the authors provide "nearly bijective" proofs of theorems about certain simplicial complexes having the same $h$-vector. There are many other cases, the theory of shifted complexes being one example, where a detour via the theory of monomial ideals has proved to be just the right approach to settling conjectures in topological combinatorics.

(4) Pertaining to the previous bullet, for certain parameter choices, the simplicial complex examined by Welker and myself is a combinatorial sphere. An important problem is to determine whether this sphere admits a realization as the boundary complex of a polytope. What makes the complex particularly interesting is that one may view it as a generalization of the associahedron. Specifically, identifying the facets of the associahedron with triangulations of polygons, the generalization for a parameter $k$ is to define the minimal nonfaces to be sets of size $k$ of pairwise crossing edges; $k = 2$ yields the associahedron. Given the multitude of polytopal realizations of the associahedron, it seems plausible that at least one of these realizations admits a generalization to our situation. I am particularly interested in examining the possibility of generalizing a recent construction due to Chapoton, Fomin, and Zelevinsky [2].



## 3 Project description

- *Theory.* My research will be based on well-established mathematical theory as alluded to in previous sections: graph theory, discrete mathematics, algebraic topology, representation theory, commutative algebra, and discrete geometry.
- *Method.* As is customary among mathematicians, my most important tools will be paper and pen. I will also use computers to check the validity of different hypotheses and to verify complicated calculations.
- *Accomplishment.* By the very nature of mathematical research, how to accomplish my goals is virtually impossible to predict in any detail whatsoever.

## 4 Significance

The four research areas discussed in previous sections are all very active. For example, there is an enormous pile of literature covering aspects of (3) intersections between combinatorics and commutative algebra and (4) geometric realizations of associahedra and related simplicial complexes. Let me discuss the two remaining areas in some detail:

(1) Over the last 15 years, there have been numerous papers and also quite a few PhD theses devoted to monotone graph properties and their relatives. The matching complex and variants thereof have been particularly well-studied. As a consequence, a major breakthrough regarding the integral homology of this complex would be of interest to a large number of mathematicians. In addition, the difficulty of understanding torsion in the homology of simplicial complexes has haunted topological combinatorialists for a long time. The matching complex seems to be a good starting point for improving our knowledge on this matter.

(2) Karp's evasiveness conjecture is considered a very important open problem in combinatorics and computer science. A proof, or a counterexample, would be considered a major achievement. In fact, any significant progress on the conjecture would be of great importance. Such progress may include results suggesting that the conjecture is true (for example, a proof of the conjecture for vertex sets of size a product of two primes) or results suggesting the opposite (for example, a counterexample to the stronger conjectures that

nontrivial monotone graph properties are noncollapsible or noncontractible).

## References

[1] S. Bouc, Homologie de certains ensembles de 2-sous-groupes des groupes symétriques, *J. Algebra* **150** (1992), 187–205.

[2] F. Chapoton, S. Fomin and A. Zelevinsky, Polytopal realizations of generalized associahedra, *Canad. Math. Bull.* **45** (2002), 537–566.

[3] J. Jonsson, *Simplicial Complexes of Graphs*, Doctoral Thesis, Kungl Tekniska högskolan, 2005.

[4] J. Jonsson, Exact sequences for the homology of the matching complex, preprint, 2006.

[5] J. Jonsson and V. Welker, A spherical initial ideal for Pfaffians, preprint, math.CO/0601335.

[6] J. Kahn, M. Saks and D. Sturtevant, A topological approach to evasiveness, *Combinatorica* **4** (1984), 297–306.

[7] C. Krattenthaler, Growth diagrams, and increasing and decreasing chains in fillings of Ferrers shapes, *Adv. Appl. Math.*, to appear.

[8] M. Rubey, Increasing and decreasing sequences in fillings of moon polyominoes, preprint, math.CO/0604140.

[9] J. Shareshian and M. L. Wachs, Torsion in the matching complex and chessboard complex, *Adv. Math.*, to appear.

VETENSKAPSRÅDET
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Kod
[redacted]
Name of applicant
Jonsson, Jakob
Date of birth
720914[redacted]

Title of research programme
Interactions between topological combinatorics and other fields of mathematics

# Appendix B

## Curriculum vitae

# 1 Doctoral degree

| | |
|---|---|
| *Year* | 2005 |
| *Field* | Topological and algebraic combinatorics |
| *University* | Kungl Tekniska högskolan, Stockholm |
| *Department* | Mathematics |
| *Title of thesis* | *Simplicial Complexes of Graphs* |
| *Advisor* | Professor Anders Björner |

# 2 Post-doctoral positions

| | |
|---|---|
| 7/2006 – 6/2007 | Instructor, Massachusetts Institute of Technology, Cambridge, Massachusetts, USA. Scientific leader: Professor Richard P. Stanley. *Planned.* |
| 3/2006 – 7/2006 | Post-doctoral position, Technische Universität Berlin, Germany. Scientific leader: Professor Günter Ziegler. |
| 1/2006 – 3/2006 | Post-doctoral position, Institut Mittag-Leffler, Djursholm, Sweden. Scientific leaders: Professor Björn Jahren and Professor Bob Oliver. |
| 7/2005 – 12/2005 | Post-doctoral position within the European Graduate Program "Combinatorics, Geometry, and Computation", Technische Universität Berlin, Germany. Scientific leader: Professor Günter Ziegler. |

# 3 –

# 4 Current employment

| | |
|---|---|
| *University* | Technische Universität Berlin |
| *Department* | Mathematics |
| *Section* | Discrete mathematics |
| *Time of employment* | March 15 – July 15, 2006 |
| *Research percentage* | 100 % |
| *Scientific leader* | Professor Günter Ziegler |

# 5 Previous employments



| | |
|---|---|
| 9/2003 – 6/2005 | Graduate student in mathematics, Kungl Tekniska högskolan, Stockholm, Sweden. |
| 9/2002 – 8/2003 | Pre-doctoral position in mathematics within the Research Training Network "Algebraic Combinatorics in Europe", Marburg University, Germany. |
| 9/1999 – 8/2002 | Part of the research staff at RSA Laboratories (the research arm of RSA Security) in Stockholm. Part time first six months. |
| 7/1997 – 3/2000 | Graduate student in mathematics, Stockholm University, Sweden. Part time last six months. |

# 6 Military service

Basic training: June 1991 – August 1992.

Refresher course: March 1997.

# 7 Awards

In 1990, I qualified as one of 18 young mathematicians for the final round of the Swedish Mathematical Olympiad.

# 8 –

# 9 –

VETENSKAPSRÅDET
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Kod
[redacted]
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Jonsson, Jakob
Date of birth
720914-[redacted]

Title of research programme
Interactions between topological combinatorics and other fields of mathematics

# Appendix C

## Complete list of publications



Vetenskapsrådet, SE-103 78 Stockholm, tel. +46 (0)8 546 44 000, vetenskapsradet@vr.se



The most important publications are marked with an asterisk.

The most relevant publications for the project under consideration are marked with two asterisks.

## 1 Refereed articles

(*) (**) Simplicial complexes of graphs and hypergraphs with a bounded covering number, *SIAM J. Discrete Math.* **19** (2005), no. 3, 633–650.

(*) (**) Generalized triangulations and diagonal-free subsets of stack polyominoes, *J. Combin. Theory, Ser. A* **112** (2005), 117–142.

(*) (**) The topology of the coloring complex, *J. Algebraic Combin.* **21** (2005), no. 3, 311–329.

(*) (**) Optimal decision trees on simplicial complexes, *Electronic J. Combin.* **12** (2005), no. 1, 31 pages.

(*) (**) On the topology of simplicial complexes related to 3-connected and Hamiltonian graphs, *J. Combin. Theory, Ser. A* **104** (2003), no. 1, 169–199.

On the number of Euler trails in directed graphs, *Math. Scand.* 90 (2002), no. 2, 191–214.

## 2 Refereed conference contributions

Securing RSA-KEM via the AES, joint work with Matt Robshaw, *Proceedings from Public Key Cryptography – PKC 2005*, Springer, 2005, 29–46.

(*) On the security of CTR + CBC-MAC, *Proceedings from Selected Areas of Cryptography – SAC 2002*, Springer, 2002, 76–93.

(*) On the security of RSA encryption in TLS, joint work with Burt Kaliski, *Advances in Cryptography – CRYPTO 2002*, Springer, 127–142.

Cryptanalysis of the NTRU Signature Scheme (NSS) from Eurocrypt 2001, joint work with Craig Gentry, Michael Szydlo, and Jacques Stern, *Advances in Cryptography – ASIACRYPT 2001*, 1–20.

Security proofs for the RSA-PSS Signature Scheme and its variants, *Second open NESSIE Workshop*, Royal Holloway, Egham, September 2001.

Funkspiel schemes: an alternative to conventional tamper resistance, joint work with Johan Håstad, Ari Juels, and Moti Yung, *ACM Conference on Computer and Communications Security 2000*, 125–133.

## 3 Theses

(*) (**) *Simplicial Complexes of Graphs*, Doctoral Thesis, Department of Mathematics, Royal Institute of Technology (KTH), Stockholm, May 2005, 350 pages.

*Euler Trails and Trees in Directed Graphs*, Licentiate Thesis, Department of Mathematics, Stockholm University, September 1999, 49 pages.

## 4 Patents

None.

## 5 Computer programs

None.

## 6 Popular scientific articles

None.

VETENSKAPSRÅDET
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Kod
[redacted]
Jonsson, Jakob
Date of birth
720914[redacted]

Dnr
2006 -

Reg date
2006-04-18 15:12:15

Project title
Interactions between topological combinatorics and other fields of mathematics

# NT

Junior research position

Mathematics & Engineering Mathematics

NT-R

Kungl Tekniska högskolan

# Appendix S

Signatures

2007 2008 2009 2010 2011

2007-07-01 -- 2011-06-30

Applicant ______________________ Date ______________________

Head of department at host University ______________________ Clarification of signature ______________________ Telephone ______________________

A signature on the application is required not only from the applicant but also from the authorised representative of the host university/institution or equivalent (normally the head of the department or establishment where the research is to be conducted). The signature confirms that the department can accommodate the project (or equipment or network), that the costing in the application is approved for the department's part, that any proposed experimentation on human or animal subjects has been reported, and that the appplicant has reported any secondary occupations and commercial ties (s)he may have, and nothing inconsistent with good research practice has thereby emerged. The applicant must have discussed these conditions with the representative of the host university/institution or equivalent before the latter approves and signs the application.

**Educational Science (not Travel and Conference grants)**
Note that the signature of an authorised representative of the host university/institution or equivalent means that the latter undertakes to invest its own resources corresponding to at least one-third of the funds obtained from the Committee for Educational Science, if the application is approved. This co-funding may include existing research if this can be linked with projects in Educational Science. Host universities/institutions or equivalent that are awarded funds must annually submit a special report on how the institution has measured up to the co-financing requirement. For research conducted in collaboration with researchers from other educational institutions, the host university/institution or equivalent is responsible for all agreements with the researchers concerned.

**International Postdoctoral Fellowships** are administered by the Swedish Research Council. The only signature required on the application is that of the applicant.

**Travel, Conference and Publication grants:** The only signature required on the application is that of the applicant.

Vetenskapsrådets noteringar
Kod
[redacted]



Vetenskapsrådet, SE-103 78 Stockholm, tel. +46 (0)8 546 44 000, vetenskapsradet@vr.se