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Optimal economic interventions in scheduled public transport☆

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ABSTRACT

This chapter analyses appropriate regulatory instruments for public transport markets under monopoly and competition, respectively. For the monopoly case, the operator chooses too low supply, compared to welfare optimum. In contrast, for the competition case the operators choose too high supply, at least for the competition model that we have considered most appropriate. It is found that under monopoly a subsidy should be applied, while under competition taxation should be applied.

1. Introduction

Is public transport characterised by market failure, so there is a reason for government intervention? If so, does intervention mean subsidisation or taxation?

We are in this study concerned solely with pure economic incentives to correct the behaviour of profit maximising operators. We disregard other possible regulatory measures, thus leaving the operators maximum freedom.

Much concern has been dedicated to public transport services, both from the general public and politicians in most countries. Should they be state owned or private? Should they be supported financially?

In most industrialised countries, the responsibility for local and regional public transport gradually became in to the hands of a Public Transport Authority, where a driving force was growing car ownership and consequently lower public transport demand and revenues. In many of these countries, the authority is also still in charge of both planning and operations of the services.

During the last two decades, another trend has flourished: deregulation. In Western Europe, this trend commenced within local and regional public transport. The privatisation of the English urban bus industry represents the "full market solution", where both supply, prices and the operation are in the hands of competing profit maximising firms. In Scandinavian countries and in London, and to some extent in other countries, the decision over local and regional public transport supply and prices has been kept in the hands of a public authority, while the actual operation is left for competition through tendering. Typically, these services need local or central government grants for financing and there is economic reason for subsidisation, partly due to the so-called Mohring effect (See Mohring, 1972), i.e., positive external effects of public transport.

The approach by Mohring has then been followed by, e.g., Jansson (1984) and Turvey and Mohring (1975) who deal with price and service frequency, assuming one passenger group. Nash (1978) optimises price and output in terms of miles operated for frequent urban bus services, contrasting maximum profit and maximum welfare solutions and assuming demand in terms of passenger miles to be dependent on price and bus miles operated. Jansson (1991) considers and contrasts frequent and infrequent services, and takes into account a variety of passenger groups. Panzar (1979) analyses infrequent airline services, assuming demand to be dependent on price and service frequency and allowing for a distribution of ideal departure times.

The other reason, for subsidisation, most relevant for urban public transport, is the second-best argument, i.e., that public transport should be subsidised when motorists do not pay the full social costs. The second-best argument is not discussed in this chapter.

In many developing countries, the typical situation is that urban public transport is both planned and operated by profit maximising firms.

For long-distance public transport, the circumstances and the economically efficient policy are less clear. Most long-distance public transport operators are commercial profit maximisers, private or state owned. Airlines, railway, coach services often compete in some ranges of distance. Is there a cause for government intervention?

Rail transport, urban and long-distance, has in most countries so far been left in the hands of governmentally controlled bodies, but two exceptions are Great Britain and Sweden. In both countries, the railway has been split into a governmental authority in charge of

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the infrastructure and one or more operating companies. These companies compete through competitive tendering, in Sweden though only for regional commuter train services.

The analysis and the results in this chapter are valid for all kinds of scheduled public transport. We analyse the scope for intervention for a monopolistic and a competitive situation, respectively. We ignore the income effect and excess burden of public financing. We find that monopolies should be subsidised and competitive operators be taxed, in order to achieve the social welfare optimum. This chapter is organised as follows.

In Section 2, we specify our basic modelling of utility and demand.

In Section 3, we study a profit maximising public transport operator who acts under monopoly, facing imperfect elastic demand. This section follows Jansson (1993) in the basic modelling approach, where price, service frequency and ride time as a function on load factor are welfare optimised, but here we also contrast welfare and profit maximisation in order to find the appropriate regulatory instrument. We find that a profit maximising operator chooses a higher price and a lower seat capacity than what a welfare maximiser would do. We find that the regulatory instrument for achieving a social optimum is to subsidise the profit maximising operator through the price paid by passengers. This subsidy can achieve a social optimum for all the relevant variables: price, service frequency and transport unit size, while a production-related subsidy cannot yield a social optimum. Some authors assume that the appropriate regulatory instrument is a subsidy related to production, see for example, Carlquist (2001) for applications in Norway. Larsen (2001) finds that both a production related subsidy and a subsidy related to the passengers should be applied. Else (1985) and Wallis and Gale (2001) on the other hand, argue that the subsidy should be related to the passengers only.

In Section 4, we study two profit maximising public transport operators who compete ("Bertrand competition") with heterogeneous transport modes. Heterogenity here stems from the difference between randomly distributed ideal departure times and actual departure times for the competing routes or modes and from randomly distributed taste for the routes or modes. Since no analytical solution to the optimisation problem is available in this case, we employ numerical calculations. The conclusion in this case is significantly different compared to the monopolistic case in Section 3. Under competition, we find that welfare optimising frequencies are below the frequencies that the operators would choose. That competition may imply too much supply from a social welfare point of view is not surprising and found in other studies; see for example, Jansson (1997) who employs a numerical example to demonstrate this. Under competition, we find that taxation instead of subsidies should be applied in order to correct a profit maximising operator. However, we cannot rule out that this result is model dependent, but we believe that the assumption of Bertrand competition is the most realistic.

In Section 5, we discuss the results from a theoretical point of view and provide examples for possible real-world applications.

2. Utility and demand

Without loss of generality, we assume that calculations of assignment, demand and consumer surplus refer to one passenger group in one origin–destination pair. This group should be as homogeneous as possible with respect to valuation of time in relation to price. For real-world analyses and applications, it is evident that passengers have to be segmented according to valuations of time and the segment specific ticket price they have to pay. We ignore the income effect, which is standard in transport analysis. All travel time components are expressed in terms of money or time by conversion with values of time.

Demand is specified for certain periods, such as the average weekday afternoon peak hour in wintertime, the average Saturday etc. Only one type of charge—a per-trip price—is considered.

The operating firm reaches decisions about relevant inputs and prices well ahead of implementation because of a necessary planning lag. All factors of production that are variable between decision and implementation are, therefore, considered relevant for the joint decision on the magnitude of policy variables. These factors include, we assume, the frequency and the size of transport unit, including the personnel required.

The generalised cost, i.e., the sum of price and travel time components expressed in monetary terms or time, for a journey from door to door has one part G common to all passengers, and one idiosyncratic part ϵ_i specific for passenger i. Thus the generalised cost for traveller i is:

$$G_i = G + \epsilon_i.$$

Each individual is assumed to have a utility of travel from origin to destination, i.e., the utility of the journey itself, which is denoted ν_i. The net utility for individual i is:

$$\nu_i - G_i = \nu_i - \epsilon_i - G.$$

Let $u_i = \nu_i - \epsilon_i$ and denote by $f[u]$ the density of u_i. Individual i chooses to travel if $u_i \geq G$, so the aggregate demand, X, is the integral of $f[u]$ above G:

$$X[G] = \int_G^\infty f[u]du. \tag{2.1}$$

Note that X is a function of G only, and that

$$X'[G] = -f[G] < 0. \tag{2.2}$$

The consumer surplus, S, is thus:

$$S = S[G] = \int_G^\infty (u - G)f[u]du.$$

It follows that

$$\frac{\partial S}{\partial G} - (G - G)f[G] - \int_G^\infty f[u]du = -X[G]. \tag{2.3}$$

3. Public transport under monopoly

3.1. Introduction

Our aim in this section is to find the appropriate regulatory instrument under monopoly. First, we compare optimal price and quality according to welfare maximisation optimum and profit maximisation optimum, respectively. Quality is here understood as wait time and ride time as a function of the load factor (seats per departure). The discrepancies between optima provide the ground for the basic aim: to find a regulatory instrument that makes the profit maximising operator behave according to welfare optimum criteria.

3.2. Notation and assumptions

We introduce the following notation:

p is the price for a trip.
s is a subsidy paid to the operator per journey.
X is the number of passengers during a period of time, thought of as 1 h.
F is frequency in number of departures per hour.

N is the number of seats per departure.
U is the product FN, i.e., the number of seats per hour.
Z is the load factor, $Z = X/U = X/(FN)$.
I is the variable infrastructure cost per departure.
e is the external cost per departure.
f is the fee paid by the operator per departure, including infrastructure costs, external costs and a possible production-related subsidy.
c is a cost proportionate to number of passengers, mainly sales costs.
r is the ride time cost; an affine or convex function of Z.
T is the cost of frequency delay (wait time).
$C[N]$ is the variable cost per departure directly related to the size of the transport unit, assumed to be an affine or convex function of N.
ϵ_p is the own-price elasticity of demand. $\partial X/\partial pp/X$,
ϵ_F is the frequency elasticity of demand.$\partial X/\partial FF/X$,
ϵ_N is the elasticity of demand with respect to number of carriages $\partial X/\partial NN/X$.

Arguments of functions are delimited by [ ], while expressions are grouped by parentheses ( ).

3.2.1. Authorities

The Public Transport Authority is assumed to be welfare maximising and responsible for infrastructure, which is subject to a user charge, f.

3.2.2. The operating firms

The operating firms may be either welfare or profit maximising. In both cases, it optimises prices, service frequency and the number of seats or carriages.

We deal with consumption efficiency related to determination of optimal prices, optimal frequency, optimal transport unit size and possible subsidies and infrastructure charges, assuming that production is efficient, irrespective of whether the actual producer is welfare or profit maximising.

3.2.3. The passengers

Aggregate demand and consumers' surplus are expressed as functions of the common generalised cost $G = $(ticket price p) + (time costs). Time costs are here assumed to comprise ride time and "frequency delay", which is the time interval between ideal and actual departure time.

The cost of ride time is:

$$r = r\left[\frac{X}{FN}\right].$$

The cost of ride time, r, is thus assumed to depend on the load factor (occupancy rate,) $Z = X/(FN)$, where r is an increasing and convex function, i.e., $r'[Z] > 0$ and $r''[Z] \geq 0$. We ignore that there are several passenger groups. This simplification will substantially facilitate the expressions derived without disturbing the purpose of the analysis (the optimal welfare maximisation conditions when various distances travelled by different passenger groups are taken into account can be found in Jansson (1991)).

The interval between departures is $1/F$ h. Ideal departure times, t, are uniformly distributed within this interval. In this section, we assume that the delay cost is a function of frequency only, i.e., $T = T[F]$. In Section 4, we will, however, consider the individual variations of delay times.

If p denotes the price, the generalised cost of travel for a group (index omitted) at time t is:

$$G[p,F,N] \equiv p + r\left[\frac{X}{FN}\right] + T[F]. \tag{3.1}$$

Note here that G is a function of occupancy rate, which is a function of demand, X, which in its turn is a function of frequency *and price*. That is, price affects demand and thus the riding time cost.

We know that own-price elasticities, denoted ϵ_p, are negative, $\epsilon_p < 0$. We assume, based on solid empirical evidence for most situations, that demand elasticities with respect to frequency and transport unit size, denoted ϵ_F and ϵ_N, are such that $0 < \epsilon_F < 1$, $0 < \epsilon_N < 1$, implying that $\partial Z/\partial F < 0$, $\partial Z/\partial N < 0$. This means that an increase in frequency or unit size will not generate so many passengers that occupancy rate is unchanged or increases.[1]

3.3. Welfare and profit optima

3.3.1. Objective functions

Below we present the objective functions of the welfare maximisation and the profit maximisation models. The maximisation relates to one service during a period normalised to 1 (h). The analysis may then be repeated for other periods and routes.

Infrastructure costs of rail operation is $I \cdot F$. The operation gives rise to external costs eF. The operators are supposed to pay the infrastructure charge $f\,F$, where not necessarily $f = I + e$. Fixed costs are not taken into account in the model since they are not affected by the operation decisions. The welfare objective function is expressed as:

$$W = S[G] + (p - c)X - F(C[N] + I + e).$$

The objective function for profit maximisation includes only producer's surplus, taking into account the infrastructure fee, f:

$$\pi = (p - c)X - F\,C[N] - fF.$$

3.3.2. Relations between welfare and profit optima

We will now examine the relations between optimal price, frequency and unit size for welfare and profit maximisation, respectively. In other words, we want to know whether price, frequency and transport unit size is larger or smaller for welfare or profit maximisation, respectively. For this purpose, we employ Topkis' theorem.[2]

We differentiate the welfare and profit functions with respect to the variables X, F and $U \equiv FN$; i.e., we transform the variables F and N into one, so that U reflects the total capacity per hour in terms of number of seats.

Of course, X is endogenous, but in the calculations below we consider X, F and U to be exogenous, and p endogenous. Thus, when we differentiate w.r.t. X, we keep U and F fixed, but let p vary. Similarly, when we differentiate w.r.t. F, we keep X and U fixed, and let p vary (so as to keep X fixed.) Note that when we differentiate w.r.t. F and U, since X is fixed, also G is fixed (cf. Eq. (2.1);) so for instance $\partial/\partial\,FS[G] = 0$.

Note that when employing Topkis' theorem, we should compare the derivatives of the objective *functions*, i.e., the comparison should be at the same point (not their respective optimum points).

In the case of no subsidy, $s = 0$, we have, employing Eqs. (2.2) and (2.3):

$$\frac{\partial}{\partial W}(W-\pi) = \frac{\partial}{\partial X}(S[G]+F(f-I-e)) = S'[G]G'[X] = -XG'[X] = -\frac{X}{X'[G]} > 0. \tag{3.2}$$

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[1] The marginal effect on occupancy rate of, for example, the frequency F is $\partial Z/\partial F = F\partial X/\partial F - X/F^2N = X(\epsilon_{F-1})/F^2N$, which is negative only if $\epsilon_F < 1$.

[2] Topkis' theorem, in the form we use it here, says that if $\partial f[x]/\partial x_i \geq \partial g[x]/\partial x_1$ and either $\partial f[x]/\partial x_i$ is increasing in x_j for all $j \neq i$, or the same is true for $g[x]$, then $\operatorname{argmax} f[x] \geq \operatorname{argmax} g[x]$.

As long as the infrastructure charge, f, is equal to, or larger than, the marginal infrastructure and external cost, we have that:

$$\frac{\partial}{\partial F}(W-\pi) = \frac{\partial}{\partial F}(S[G]+F(f-I-e)) = f-I-e \geq 0. \tag{3.3}$$

Next, we differentiate with respect to U:

$$\frac{\partial}{\partial U}(W-\pi) = \frac{\partial}{\partial U}(S[G]+F(f-I-e)) = 0. \tag{3.4}$$

We are now almost prepared to conclude that at welfare optimum, X, F and U is greater (or possibly equal) to the situation with profit maximisation. The first condition in Topkis' theorem[2] is satisfied, as is seen from Eqs. (3.2), (3.3), and (3.4). However, we must also check the cross effects for one of the objective functions, and we choose W. First, we compute $\partial W/\partial F$:

$$\frac{\partial W}{\partial F} = \frac{\partial p}{\partial F}X - C[N] + NC'[N] - I - e.$$

In order to compute p_F, we note that by Eq. (3.1):

$$0 = \frac{\partial G}{\partial F} = \frac{\partial p}{\partial F} + T'[F]$$

i.e.,

$$\frac{\partial p}{\partial F} = -T'[F].$$

Hence,

$$\frac{\partial W}{\partial F} = -XT'[F] - C[N] + NC'[N] - I - e.$$

Now we can compute $\partial^2 W/\partial F \partial X$ and $\partial^2 W/\partial F \partial U$:

$$\frac{\partial^2 W}{\partial F \partial X} = -T'[F] > 0,$$
$$\frac{\partial^2 W}{\partial F \partial U} = \frac{N}{F}C''[N] \geq 0.$$

The computation of $\partial W/\partial U$ is similar to that of $\partial W/\partial F$:

$$\frac{\partial W}{\partial U} = \frac{\partial p}{\partial U}X - C'[N].$$

Here $\partial p/\partial U$ can be computed in the same way as $\partial p/\partial F$:

$$0 = \frac{\partial G}{\partial U} = \frac{\partial p}{\partial U} - r'\left[\frac{X}{U}\right]\frac{X}{U^2},$$

i.e.,

$$\frac{\partial p}{\partial U} = r'\left[\frac{X}{U}\right]\frac{X}{U^2},$$

hence,

$$\frac{\partial^2 W}{\partial U \partial X} = \frac{\partial}{\partial X}\left(r'\left[\frac{X}{U}\right]\frac{X^2}{U^2} - C'[N]\right) = r''\left[\frac{X}{U}\right]\frac{X^2}{U^3} + 2r'\left[\frac{X}{U}\right]\frac{X}{U^2} > 0.$$

To conclude: *The profit maximiser has too few passengers and too low frequency from a welfare point of view.* He has also lower capacity FN than the welfare maximiser; however, given the fewer passengers, this may or may not be sub-optimal.

3.3.3. Welfare and monopoly equilibria

The first order conditions referring to price, frequency and unit size for a welfare optimiser are, respectively,

$$0 = \frac{\partial W}{\partial p} = -X\frac{\partial G}{\partial p} + (p-c)X'[G]\frac{\partial G}{\partial p} + X \tag{3.5a}$$

$$0 = \frac{\partial W}{\partial F} = -X\frac{\partial G}{\partial F} + (p-c)X'[G]\frac{\partial G}{\partial F} - (C[N]+I+e) \tag{3.5b}$$

$$0 = \frac{\partial W}{\partial N} = -X\frac{\partial G}{\partial N} + (p-c)X'[G]\frac{\partial G}{\partial N} - FC'[N]. \tag{3.5c}$$

The first order conditions referring to price, frequency and unit size for a monopolist are, respectively,

$$0 = \frac{\partial \pi}{\partial p} = (p-c)X'[G]\frac{\partial G}{\partial p} + X \tag{3.6a}$$

$$0 = \frac{\partial \pi}{\partial F} = (p-c)X'[G]\frac{\partial G}{\partial F} - C[N] - f \tag{3.6b}$$

$$0 = \frac{\partial \pi}{\partial N} = (p-c)X'[G]\frac{\partial G}{\partial N} - FC'[N]. \tag{3.6c}$$

Note that it is impossible to achieve the welfare optimum by adjusting f away from $I+e$. This is seen by a *reductio ad absurdum*: assume that we *can* achieve welfare optimum by the profit maximiser by a suitable value of f. It then follows that Eqs. (3.5a) and (3.6a) are evaluated at the same values of p, F and N, and hence that $XG_p = 0$ at this point, which is clearly absurd. Hence, one cannot apply a subsidy only related to frequency in order to achieve a welfare optimum. If the monopolist enjoys a subsidy s of the price, his objective function is

$$\pi = (p+s-c)X - FC[N] - fF,$$

hence, the first order conditions become

$$0 = \frac{\partial \pi}{\partial p} = (p+s-c)X'[G]\frac{\partial G}{\partial p} + X \tag{3.7a}$$

$$0 = \frac{\partial \pi}{\partial F} = (p+s-c)X'[G]\frac{\partial G}{\partial F} - C[N] - f \tag{3.7b}$$

$$0 = \frac{\partial \pi}{\partial N} = (p+s-c)X'[G]\frac{\partial G}{\partial N} - FC'[N]. \tag{3.7c}$$

We see that if the subsidy s is set to $S = -X/X'[G]$ at the welfare optimum and $f = I+e$, then the three Eqs. (3.5a), (3.5b), and (3.5c) coincide with Eqs. (3.7a), (3.7b), and (3.7c), and hence *the profit optimum coincides with the welfare optimum. The passenger related subsidy can thus yield the social optimum with respect to all the relevant policy variables: price, frequency and transport unit size.* The expression for the subsidy s can be expressed in various ways:

$$s = -\frac{X}{X'[G]} = -\frac{G}{\epsilon_G} = -\frac{p}{\epsilon_p} - r'[Z]Z$$

where p is the price of the welfare optimiser. The last equality follows from the fact that $\partial G/\partial p = 1 + r'[Z]Z\epsilon_p/p$.

If, for example, the elasticity with respect to price would be around -1, the subsidy would be equal to the chosen price minus a term that depends on the marginal congestion cost. Wallis and

Gale (2001) find, like us, that the passenger related subsidy is $-G/\epsilon_G$, but also that the subsidy can be expressed as $-p/\epsilon_p$. Our result includes an additional term related to the marginal congestion cost, this, since we explicitly take this quality variable into account.

Else (1985) finds an optimal subsidy related to passengers, but which differs from ours.

4. Public transport under competition

When dealing with competition between operators or modes a crucial issue is what factors affect the passengers' choice. Clearly travel time components and price matter, among other factors, and time and price are not valued the same by all individuals. These are important facts that should not be ignored.

There are various ways that one can take care of variations of travel time and price, as well as other factors: Apply randomness for taste variations within each group, segment passenger groups with different values of travel time, i.e., take taste into account in this way, apply randomness to the passengers' different ideal departure or arrival times.

A popular way to model taste variation is the so-called logit model (belonging to the extreme value family). Here we have chosen a different approach though, since we believe the logit model has shortcomings when applied to public transport[3].

We, basically, take into account randomness with respect to the difference between passengers' ideal departure or arrival times, bearing in mind that the analysis can be repeated for a number of segments.

4.1. Basic micro-economic model

In Section 3, we defined generalised cost as:

$$G[p,F,N] = p + r\left[\frac{X}{FN}\right] + T[F].$$

In this section, we ignore that ride time cost may depend on in-vehicle congestion, and call $p+r$ travel cost R. That is, $R=p+r$. We also ignore the size of the vehicle, N. Instead, we take into account that the wait time is variable, i.e., dependent on t, the difference between ideal departure or arrival time and actual departure or arrival time. We model competition by using the fact that the passengers will choose the competitor that has the smallest total travel cost $R+t$.

We assume that the individual differences between ideal and actual departure or arrival times are randomly, uniformly distributed.

Each travel alternative in a specific origin–destination pair has a total travel cost and each passenger chooses the alternative with minimum total cost. In order to simplify notation and calculations, we assume that there are only two alternatives, 1 and 2.

The total cost of alternative j ($j=1, 2$) for each individual i is the sum of travel cost R^j (including all travel time components plus price, except wait time) and a random variable, t_i^j, the time between ideal arrival time and actual arrival time. With a valuation of time, wait time, ride time, and transfer time can all be expressed in monetary units, or conversely monetary units can be thought of as measures of time. In the following, all quantities are measures of time but we may, and often do, describe them as costs.

When each individual chooses the alternative with the minimum total cost, the effective cost becomes:

$$\min\left[R^1 + t_i^1 + R^2 + t_i^2\right].$$

We now assume that (t^1, t^2) is uniformly distributed on $[0, H^1]\times[0, H^2]$, where headway $H^i = 1/F^i$. This, since we have no knowledge of the true distribution of ideal departure or arrival times for the period of time (peak hours or non-peak hours for example) we are analysing.[4]

Notation

H^1	headway of route 1.
H^2	headway of route 2.
R^1	travel time (including price expressed in minutes) of route 1.
R^2	travel time (including price expressed in minutes) of route 2.
t^1	time to departure of route 1.
t^2	time to departure of route 2.

The probability for choosing alternative 1 is then:

$$\Pr[1] = \frac{1}{H^1H^2}\int_0^{H^1}\int_0^{H^2} h\left[R^2 - R^1 + t^2 - t^1\right]\mathrm{d}t^2\mathrm{d}t^1 \tag{4.1}$$

where $h[s]$ is the Heaviside function, defined by:

$$h[s] = \begin{cases} 1 & \text{if } s \geq 0 \\ 0 & \text{if } s < 0 \end{cases}.$$

The model thus assumes that passengers know the timetable and choose route, stop and mode, taking into account all travel time components and price and how well ideal departure times relate to actual departure times.

Here, the expected wait time, T, can be expressed as:

$$T = \frac{1}{H^1H^2}\int_0^{H^1}\int_0^{H^2} h\left[R^2 - R^1 + t^2 - t^1\right]\left(t^1 - t^2\right) + t^2\mathrm{d}t^2\mathrm{d}t^1.$$

In general, if there are k acceptable routes and the travel cost for route j is R^j and the probability of choice of route j is denoted $\Pr[j]$, the expected travel time R, and the expected wait time T are given by:

$$R = \sum_{j=1}^{k}\Pr[j]R^j, \quad T = \sum_{j=1}^{k}\Pr[j]E[t|j] \tag{4.2}$$

respectively, where $E[\cdot|j]$ denotes expectation conditioned on route number j being chosen.

The probability in expression Eq. (4.1) and the conditional expectation $E[t^j|j]$ used in Eq. (4.2) can be calculated explicitly. This is illustrated when having two alternative modes. The first route is selected when $R^1+t^1 < R^2+t^2$, that is when $t^2 > t^1 + (R^1 - R^2)$. For different values of $d = R^1 - R^2$, the probabilities of $t^2 > t^1 + d$ and the conditional expectations are given by:

(i) if $d > H^2$ then

$$\Pr[1] = 0$$

[3] See Jansson et al. (2008).

[4] The result that follows for choice probability and expected wait time has also been presented, without specific derivation, by Hasselström (1981). The principles for taking into account randomness of ideal departure times have been implementd in the public transport network softwares Vips and Visum.

$E\left[t^1|1\right]$ is undefined

$$\mathrm{E}\left[t^2|2\right] = \mathrm{H}^2/2,$$

(ii) if $H^2 > d > \max\,[H^2 - H^1, 0]$ then

$$\Pr[1] = \frac{\left(H^2 - d\right)^2}{2H^1H^2}$$

$$E\left[t^1|1\right] = \frac{H^2 - d}{3}$$

$$E\left[t^2|2\right] = \frac{d^3 - 3d\left(H^2\right)^2 - H^2\left(3H^1H^2 - 2\left(H^2\right)^2\right)}{3\left(d^2 - 2dH^2 - 2H^1H^2 + \left(H^2\right)^2\right)},$$

(iii) if $\max\,[H^2 - H^1, 0] > d > 0$ then

$$\Pr[1] = 1 - \left(H^1/2 + d\right)/H^2$$

$$E\left[t^1|1\right] = \frac{3dH^1 + H^1\left(2H^1 - 3H^2\right)}{3\left(2d + H^1 - 2H^2\right)}$$

$$E\left[t^2|2\right] = \frac{3d^2 + 3dH^1 + \left(H^1\right)^2}{3H^1 + 6d},$$

(iv) if $0 > d > -\max\,[H^1 - H^2, 0]$ then

$$\Pr[1] = \left(\mathrm{H}^2/2 - d\right)/H^1$$

$$E\left[t^1|1\right] = \frac{3d^2 - 3dH^2 + \left(H^2\right)^2}{3\left(H^2 - 2d\right)}$$

$$E\left[t^2|2\right] = \frac{-3dH^2 + H^2\left(2H^2 - 3H^1\right)}{3\left(-2d + H^2 - 2H^1\right)},$$

(v) if $-\max\,[H^1 - H^2, 0] > d > -H^1$ then

$$\Pr[1] = 1 - \frac{\left(H^1 + d\right)^2}{2H^1H^2}$$

$$E\left[t^1|1\right] = \frac{-d^3 + 3d\left(H^1\right)^2 - H^1\left(3H^1H^2 - 2\left(H^1\right)^2\right)}{3\left(d^2 + 2dH^1 - 2H^1H^2 + \left(H^1\right)^2\right)}$$

$$E\left[t^2|2\right] = \frac{H^1 + d}{3},$$

(vi) if $-H^1 > d$ then

$$\Pr[1] = 1$$

$$E\left[t^1|1\right] = H^1/2$$

$E\left[t^2|2\right]$ is undefined.

Below we provide an example concerning the choice between two alternatives. We assume that the headways for both routes 1 and 2 are 15 min, i.e., $H^1 = H^2 = 15$, but the travel times for the routes differ; for route 1 it is $R^1 = 15$ min, for route 2 it is $R^2 = 20$ min.

Route 1 is selected if $R^1 + t^1 \le R^2 + t^2$, where t^1 and t^2 are independent and uniformly distributed on $[0, H^1]$ and $[0, H^2]$, respectively. Hence, if $t^1 \le 5$ then:

$$\Pr\left[1\middle|t^1\right] = \Pr\left[t^2 \ge \underbrace{R^1 - R^2 + t^1}_{\le 0}\middle|t^1\right] = \Pr\left[t^2 \ge 0\middle|t^1\right] = 1.$$

Furthermore, when $5 \le t^1 \le 15$ then:

$$\Pr\left[1\middle|t^1\right] = \Pr\left[t^2 \ge \underbrace{R^1 - R^2 + t^1}_{\ge 0}\middle|t^1\right] = 1 - \frac{R^1 - R^2 + t^1}{H^2} = \frac{20 - t^1}{15}.$$

This gives,

$$\Pr[1] = \int_{-\infty}^{\infty} \Pr\left[1\middle|t^1\right] f\left[t^1\right] dt^1 = \int_0^{15} \Pr\left[1\middle|t^1\right] \cdot \frac{1}{15} dt^1 = \int_0^5 1 \cdot \frac{1}{15} dt^1 + \int_5^{15} \frac{20 - t^1}{15} \cdot \frac{1}{15} dt^1 = \frac{1}{3} + \frac{4}{9} = \frac{7}{9}.$$

Fig. 1 below illustrates points of time, headway and travel times of the two routes. The total bar lengths represent the maximal costs associated with the routes. We assume that the two routes arrive at the same time (0). The passengers' ideal departure times are along the x-axis; the wait time for each alternative is a uniformly distributed random variable over the light coloured parts of the bars. The passengers chose the alternative where this wait time variable is closest to the origin, having smallest total cost. For ideal departure times between 15 and 20 min only route 1 is chosen. For other times the passengers are split between the two alternatives according to the proportions $\Pr[1|t^1]$ and $1 - \Pr[1|t^1]$.

The conditional probability of selecting route 1 is shown in Fig. 2.

Let u denote the net utility, measured in pecuniary terms, of a journey for an individual. The total consumer surplus S for travel is then:

$$S = \frac{X}{H^1H^2} \int_0^{H^2} \int_0^{H^1} u - \min\left[R^1 + t^1, R^2 + t^2\right] dt^1 dt^2 = X \int_0^1 \int_0^1 \max\left[u - R^1 - H^1\tau^1, u - R^2 - H^1\tau^2\right] d\tau^1 d\tau^2.$$

Since $\partial/\partial x \max[x, y] = h[x - y]$ we have

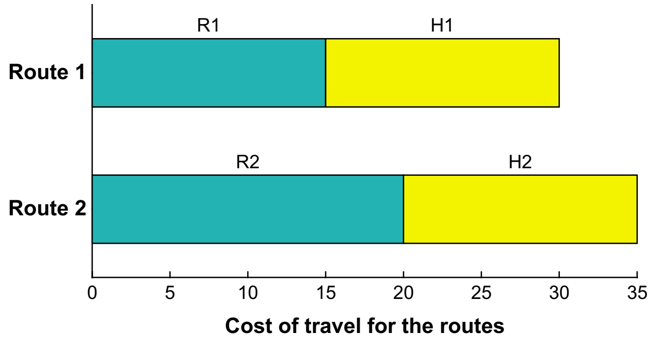

Fig. 1. Travel time and headway for routes 1 and 2. For each alternative the wait time is a random variable uniformly distributed over the yellow (light coloured) parts of the bars.

$$\frac{\partial S}{\partial R^1} = -X\int_0^1\int_0^1 h\left[R^2+H^2\tau^2-R^1-H^1\tau^1\right]d\tau^1 d\tau^2 = -X\cdot\Pr[1] = -X^1$$

and

$$\frac{\partial S}{\partial H^1} = -X\int_0^1\int_0^1 \tau^1 h\left[R^2+H^2\tau^2-R^1-H^1\tau^1\right]d\tau^1 d\tau^2 = -\frac{1}{H^1}E\left[t^1|1\right]X^1.$$

Note that $E[t^1|1]$ is shorter than $H^1/2$, since there is another alternative! Hence, in order to compute the change ΔS in consumer surplus due to a change ΔR of R^i, one can integrate the demand function X^i for mode i:

$$\Delta S = -\int_{R^i}^{R^i+\Delta R} X^i \mathrm{d}R^i$$

and similarly for a change in headway H^i by ΔH:

$$\Delta S = -\frac{1}{H^i}\int_{H^i}^{H^i+\Delta H} X^i E\left[t^i|i\right]\mathrm{d}H^i. \qquad (4.3)$$

Note, however, that in Eq. (4.3) we must take into account that $E[t^i|i]$ depends on what other alternative travel modes there are, so the mode i cannot be considered in isolation.

4.1.1. Demand

We consider two competing firms providing public transport meeting a fixed total demand. The modes of transport differ in some respect, i.e., one firm may transport by air and another by railway. We denote the two transport firms 1 and 2. The basic model is now extended to include a taste variation. Each traveller has a taste parameter θ which is interpreted as a disutility (measured in pecuniary measure, i.e., a "cost") for travel: θ is the disutility for travel with 1, and $L-\theta$ the disutility for travel with firm 2. The taste parameter θ over individuals is assumed to be uniformly distributed in the interval $0\le\theta\le L$.

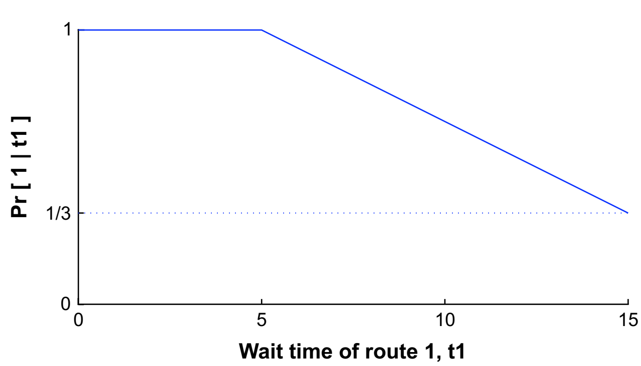

Fig. 2. The conditional probability of selecting route 1 as a function of t^1.

Each traveller has also an ideal departure time, and the difference between the ideal departure (arrival) time and the most convenient actual departure (arrival) time of firm i is denoted t^i. Furthermore, the travel time and ticket price for firm i are denoted r^i and p^i. With these parameters, the total travel cost R^i is given by $R^i = r^i + p^i$. The traveller will chose the firm offering the lowest generalised cost: he will chose firm 1 if

$$p^1+r^1+t^1+\theta < p^2+r^2+t^2+L-\theta$$

and firm 2 otherwise.

The random variables t^1, t^2, and θ are assumed to be uniformly distributed on $[0, H^1]\times[0, H^2]\times[0, L]$. Here H^j is the headway of firm j.

We can now express the demand for the two firms. There is a total cohort of X individuals, all of whom will travel with either firm. The number of travellers choosing firm 1 is

$$X^1 = \frac{X}{LH^1H^2}\int_0^L\int_0^{H^2}\int_0^{H^1} h\left[\left(p^2+r^2+t^2+L-\theta\right) - \left(p^1+r^1+t^1+\theta\right)\right]\mathrm{d}t^1\mathrm{d}t^2\mathrm{d}\theta$$

where $h[x]$ is again the Heaviside function. The demand for firm 2 is obviously $X^2 = X - X^1$.

4.1.2. Welfare

In order to study the welfare effects, we have to compute the welfare function. In this case, with inelastic demand, it is convenient to calculate welfare costs, i.e., we set the travellers' utility of travel to zero (this number will not affect welfare *changes*) and compute the negative of the welfare.

The total welfare costs for the travellers is:

$$C_c = \frac{X}{LH^1H^2}\int_0^L\int_0^{H^2}\int_0^{H^1}\min\left[p^1+r^1+t^1+\theta, p^2+r^2+t^2+L-\theta\right]\mathrm{d}t^1\mathrm{d}t^2\mathrm{d}\theta.$$

The total welfare cost is then

$$W_c = C_c - \text{profit of firm1} - \text{profit of firm2} + \text{external costs (pollution, noise, etc.)}$$

4.2. Competition

There are three common models of competition in the literature. The *Bertrand model*, the *Cournot model* and the *Stackelberg model*. In the *Bertrand model*, each firm takes the other firms' prices as given (fixed,) and maximise their own profit under that assumption by choosing the relevant price. That is, each firm's price is optimal (i.e., profit maximising) given the other firms' prices. The equilibrium outcome is the *Bertrand equilibrium*. The Bertrand competition is very strong; if all firms produce the same good (perfect substitutes,) then the price is driven down to marginal cost, even with only two firms in the market. If firms have different marginal costs, only the firm(s) with minimum marginal cost(s) will stay in the market. Therefore, one usually assumes that products are imperfect substitutes, so that an equilibrium with prices above marginal cost results.

No model, including *Stackelberg* and *Cournot*, is perfectly convincing intuitively—indeed, they produce differing equilibria, so they cannot all prevail at the same time. In the literature, it is our impression that the Bertrand model is the most favoured when products are differentiated (imperfect substitutes), and since transport modes are heterogeneous, this is the model we choose in this study.

In order to compute the Bertrand equilibrium values of p^1, p^2, H^1 and H^2, we first consider firm 2's decision problem, given some (arbitrarily chosen) values of p^1 and H^1; he maximises profit:

$$\underset{H^2,p^2}{\operatorname{argmax}}\, X^2\left[p^1,p^2,H^1,H^2\right]\left(p^2-c^2\right)-C^2\left[H^2\right]$$

where X^2 is the demand for the given values of parameters, c^2 is the cost per passenger, and $C^2[H^2]$ the cost associated with the departure interval H^2. The outcome produces firm 2's price and headway as functions of the corresponding values chosen by firm 1:

$$\left(p^1,H^1\right)\mapsto\left(p^2,H^2\right) \tag{4.4}$$

Now the decision problem of firm 1 is similar; he maximises his profit, given p^2 and H^2:

$$\underset{H^1,p^1}{\operatorname{argmax}}\, X^1\left[p^1,p^2,H^1,H^2\right]\left(p^1-c^1\right)-C^1\left[H^1\right].$$

The outcome produces firm 1's price and headway as functions of the corresponding values chosen by firm 2:

$$\left(p^2,H^2\right)\mapsto\left(p^1,H^1\right). \tag{4.5}$$

Now we have to find values of p^1, p^2, H^1 and H^2, such that both Eqs. (4.4) and (4.5) are satisfied simultaneously. Technically, we do this by fixed point iteration: we start with some initial values of p^1 and H^1, compute the resulting values of p^2 and H^2 according to Eq. (4.4), use these values in Eq. (4.5) to compute new values of p^1 and H^1, use these values in turn to compute new values of p^2 and H^2 employing Eq. (4.4), and so on, iteratively, until the values converge.

4.3. Numerical evaluation and results

This section contains results obtained by numerical computation of Bertrand equilibrium for competing public transport services, as well as numerical values of minimal welfare cost for such competitors. Several computations are performed to reflect how the equilibrium and obtained welfare vary when fundamental parameters change.

The basic set of parameter values are chosen to resemble the situation for train and airline operators in Sweden, specifically on the route between Stockholm and Gothenburg, the capital and the second largest city. The numbers seem reasonable according to information from the Swedish National Rail Administration (Banverket) and literature on airline costs. The external costs are those normally applied by Banverket and the Swedish Institute for Transport and Communications Analysis (SIKA). We also use the infrastructure charges applied in Sweden.

The total demand is 4000 travellers per day. The taste parameter (the disutility) is uniformly distributed on the interval (0, 500) SEK, i.e., some passengers are willing to pay up to 500 SEK extra to avoid a particular operator. The parameters of importance are presented in Table 1 below.

Table 2 summarises the effect of different settings of the policy variables on the frequencies of the operators. From the table one finds that a low tax (or high subsidy) yields operating frequencies for both operators that are higher than what is optimal from a welfare perspective. Conversely, a high tax yields too low frequencies. The infrastructure charge behaves similarly.

The competition with an airline operator made the train operate at a frequency higher than optimal regardless of the value of travel time or wait time. For high values of the travel/wait time the airline operated with too low frequencies though.

The basic results are as follows. Since we assume a constant total demand, the welfare is unaffected by the level of prices as long as their difference is the same. We can thus compute the welfare optimising price *difference*, but not a level. We find that under competition, this price difference is the same as under welfare optimisation, but that welfare optimising frequencies are *below* the frequencies that the operators would choose.

The last result can be explained in terms of negative externalities: when one firm increases its frequency, it imposes a *negative externality* on the other firm, in that it decreases that other firm's demand. Since this cost is not internalised by the first firm, it chooses a higher frequency than is optimal.

4.3.1. The numerical procedure

The numerical computations are performed as follows. When calculating the Bertrand equilibrium, an operator's action and the competitor's reaction are calculated successively until the equilibrium is reached. The equilibrium thus obtained is a fix point. In each step of the iteration, a competitor's reaction is determined by searching a grid of price and interval pairs for the values that maximise the profit of the operator. As the algorithm converges, the search is made on a finer and finer grid, allowing the optimal price and interval to be accurately determined.

The minimal welfare cost is determined by an exhaustive search on a grid of distinct interval lengths for the operators and their price difference. The welfare optimum is the particular triple of interval lengths and price difference that minimise the welfare cost. The search is then performed repeatedly on a finer parameter grid, centered on the previously obtained parameter triple, until the optimum is determined with sufficient accuracy.

Table 1
Assumed fixed and variable parameters.

	Fixed parameters				
	Costs		Distance (km)	Travel time (h)	External Costs (SEK/km)
	Operating (SEK/km)	Passengers (SEK/pass)			
Rail	163	40	455	4.50	1.30
Air	211	40	394	3.50	8.00

	Variable parameters					
	Values of time		Ride time weight	Infrastructure charge (SEK/km)	Demand No of passengers	Taste parameter Max (SEK/journey)
	Travel (SEK/h)	Wait (SEK/h)				
Rail	100	50	1.00	5.00		500
Air	110	55	1.10	0.36		500
Total					4,000	

Table 2
An overview of how different settings of the policy variables affect the frequencies of the operators.

	Low		High	
	Train	Air	Train	Air
Tax	+	+	−	−
Infrastructure charge	+	+	−	−
Value of travel time	+	+	+	−
Value of waiting time	+	+	+	−
Demand	+	−	+	+
Taste/disutility	+	−	+	−

A '+'-sign indicates that the frequency is higher than optimal for the operator and parameter setting, conversely a '−'-sign indicates a too low frequency.

In these numerical computations, the headways, H^1 and H^2, are not allowed to be greater than 16 h, thus bounding the maximal time interval between departures, or putting a lower bound on the frequency of departures. When a time interval for a competitor is set to 16 h by the computer programme, it usually signifies that the actual (optimal) interval is greater, possibly infinite.

4.3.2. Effects of tax/subsidy on departures

A subsidy or tax on prices did nothing but change the ticket price with the corresponding amount. The operators did not change their behaviour and consequently did not come any closer to an optimal welfare cost. This is an artifact of two model specifications: (1) the total demand is inelastic and (2) the demand for the individual firms depend only on the price difference (Fig. 3).

A lowered subsidy or increased tax on departures when having two equal (train) operators decreased the frequency but did not affect the ticket price (cf. subsidy or tax on prices). The profits decreased as the taxes increased. The optimal welfare cost is not affected by taxes but the obtained welfare is, and the least difference between obtained and optimal welfare costs is achieved when the tax is 10–20 SEK/km higher per departure than the current tax level.

For two *different* operators, a lowered subsidy or increased tax decreases the frequencies. Again, the least difference between obtained and optimal welfare costs is achieved when the tax is increased by 10–20 SEK/km per departure from the current tax level.

4.3.3. Infrastructure charge

As an alternative to add taxes/subsidies relative to the current level of taxation, one may subject every mode to an infrastructure charge, equal to the external costs of the mode. This makes the modes internalise the external costs. In this setting, the infrastructure charge is varied around the zero level where the external costs are fully compensated.

Changing the infrastructure charge does not affect the ticket price when having two equal operators, similar to the behaviour with tax on departures. Profits decrease when the charge is increased. The optimal welfare cost is unaffected by any such charge, and the minimal difference between obtained and optimal welfare costs is achieved when there is a charge around 20 SEK/km and departure.

For two different operators, an increased charge decreased the frequencies. Again, the least difference between obtained and optimal welfare costs is achieved when the charge is around 20 SEK/km per departure.

4.3.4. Value of travel time

When having two equal operators, larger travel time values give higher welfare costs, both under competition and optimality, but constant difference between them. The ticket price and frequency of operation were not affected by the changes in travel time values, and thus, the profits of the operators are constant.

The optimal welfare, having both an air and a train operator and travel time values above 260 SEK/h, is obtained if the train does not operate. Under competition, the ticket price decreases for train but increases for air operators, as the travel time value increases.

The total cost of travel increases with larger travel time values. The difference in total cost of travel between operators is zero when the travel time value is 20 SEK/h. When the travel time value is larger than 20 SEK/h then the obtained total price difference between the operators is smaller than the optimal price difference. The frequency of trains decreases but for flights increases, as the travel time value increases. Under competition the train operates more frequently than what is optimal, but the airline less, at least

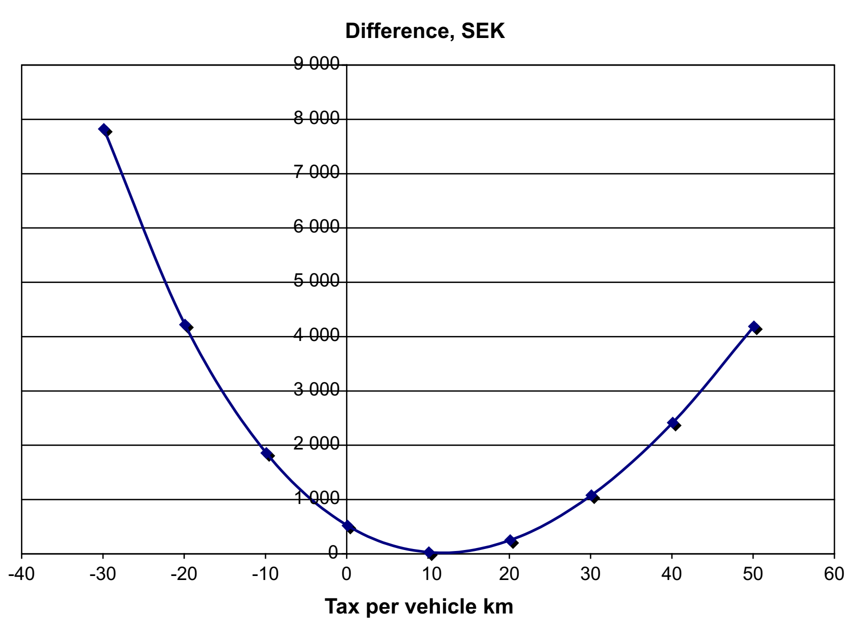

Fig. 3. Illustration of the difference between the welfare obtained by the profit maximiser and the welfare maximiser for various subsidy levels.

for large values of the travel time value. For small travel time values, even the airline is too frequent.

The welfare cost decreases as the travel time value decreases. The difference between obtained and optimal welfare cost is at a minimum when the travel time value is 30 SEK/h.

4.3.5. Value of wait time

Generally, when the wait time value is small, the competitors operate with a low frequency, i.e., they have long intervals between departures. As the wait time value increases, so does the frequency for both operators. The welfare cost increases with the wait time value and also the difference between obtained welfare cost under competition and minimal welfare cost, determined by optimisation. We note that the intervals obtained under competition are shorter than the corresponding intervals under welfare optimisation.

An analysis of a competition between two equal operators (train services) reveals that when the value of the wait time is large, i.e., it is expensive to wait, it is not optimal to have two operators. That is, for wait time values larger than 230 SEK/h, say, it is better in a welfare sense to have a single firm operate at, more or less, the double frequency.

The analysis of the competition between a train and an airline operator exhibits similar behaviour. When having large wait time values (150 SEK/h or more) it is optimal to have only an airline operator. It is worth noting that, under optimality, the difference in price between the operators is constantly 65 SEK, irrespective of the wait time value. The 65 SEK is the difference in travel time value, meaning that optimally the operators should have the same ticket price.

4.3.6. Demand

The ticket price for two competing train operators decreases as the demand increases, but never falls below 540 SEK. The frequency increases as the demand increases, and the difference between obtained and optimal welfare cost decreases in absolute numbers.

It is worth noting that for very small demand it is probably not optimal to run any services at all. When having two different operators and a demand in the range from 600 to 1000 travellers per day, it is optimal in the welfare cost sense to only run a single airline operator. The ticket price for the train is constantly decreasing while the ticket price for the airline company has a minimum when the demand is around 1100–1300 passengers per day. As the demand increases, so does the frequency, and when it is optimal to have two operators, both operate too frequent.

4.3.7. Taste/disutility

The welfare cost increases as passengers are more willing to pay for avoiding a particular operator. The difference between obtained and optimal welfare cost decreases with the willingness to pay.

Two equal operators: the frequencies are not affected by a change in the taste parameter, but the prices increase when the willingness to pay increases, hence, the profits increase. If the willingness to pay is small, it is optimal to have only a single operator.

Two different operators: the prices (and profits) increase but the difference in frequency diminishes; a train increases the number of departures, an airline lowers the frequency. Yet, when the taste parameter is small the train runs too frequent and the airline too seldom.

4.3.8. Comfort

A measure of comfort is the time value of a mode. High time values indicate that passengers are willing to pay a lot for a reduction of time, i.e., the mode is uncomfortable. The ratio of time values for airline and train is 1.1 in the standard setting; signifying that travelling by airplane is 10% more uncomfortable than by train, in a sense.

Varying the ratio between the time values for airline and train shows that small values (train uncomfortable compared to airplane) give a welfare optimum with no train, and high values a welfare optimum with no airline. Only for values of the ratio in the range [0.9, 1.4] optimality is obtained with two operators. When the airplane is 20% more uncomfortable than the train (ratio 1.2), the operators split the demand in two equal parts.

5. Discussion

We have found that profit maximisng public transport operators should be subsidised if they are monopolies, but they should be taxed if they operate in competitive markets. The fact that a monopoly charges too high a price comes as no surprise; maybe less obvious is the fact that the regulatory instrument is a subsidy upon the pecuniary price the operator chooses, which makes the operator choose welfare optimal price and frequency as well as transport unit size. We believe that this result is reasonably robust to model specifications.

The situation is different under competition, where we find that supply tend be too high. If one operator increases the frequency of departures, the competitor will suffer a loss as a consequence of fewer passengers, and this negative externality causes the equilibrium to show too high frequencies. In order to come closer to welfare optimum, a tax per departure is suggested. It is, however, a common experience that comparative statics results in competition models can be very sensitive to model specifications, in particular to the mode of competition (competition in prices versus supply,) so further investigation of models with other modes of competition is called for.

In some cases, demand is too low to sustain more than one operator. For instance, if travel time values are high, people will prefer the fast mode, air instead of rail, for example, and there may be no room for the slow mode. So, according to our modelling, when a competitive situation shifts to monopolistic situation, also the optimal policy shifts from taxation to subsidisation.

What about real-world examples and possible applications?

For local and regional transport, the situation where a private profit maximising firm virtually has monopoly, like in many cities in Great Britain, a passenger related subsidy may be considered.

On the other hand, in cities where there is real on-road competition, like in many developing countries and in some places in Great Britain for example, taxation may be considered.

With respect to long-distance public transport, we can give two examples from Scandinavia.

There are many long-distance transport markets where competition is fierce. One example is Stockholm centre – Gothenburg centre in Sweden where there are three airlines (1 h plus access time), one railway operator (3 h) and several coach operators (6 h.) The supply here may be too large and inefficiently costly. Taxation would reduce the number of departures and maybe also squeeze out some operators. One would achieve resource saving that increases overall welfare.

In many places in the sparsely populated Norway, for example, mountains, islands and fjords make rail-, coach- and car transport very expensive or impossible. Only air transport is efficient. In these cases, most of the origin–destinations have only one operating airline. The simple reason is that more than one operator could not survive due to the low demand. Consequently prices are very high and should be subsidised, which they actually also are to some extent.

With respect to both urban and long-distance transport where competitive tendering for operation under gross-cost contract is applied, one may consider subsidisation of ticket prices under net-cost contract as an alternative.

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