Perfect codes and related topics
(Open Problems)

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First challenging open problem:

Find the classification of all perfect q-ary codes of length n over the Galois field $GF(q)$.

The solution for the binary case $n = 15$ is done in [1].

REFERENCES

[1] Östergård, P. R. J. and Pottonen, O., The perfect binary one-error-correcting codes of length 15: Part I—Classification, *IEEE Trans. Inform. Theory* **55** (2009), 4657–4660.

Author: Danyo Danev
Title: Families of quasi-perfect codes

Some definitions:

- $GF(q) \longrightarrow$ the Galois field of q elements, where $q = p^s$;
- $H(n,q) \longrightarrow$ the Hamming space : $\{\mathbf{a}=(a_0,\ldots,a_{n-1}) : a_i \in GF(q)\}$;
- $d_H(\mathbf{x},\mathbf{y}) = d(\mathbf{x},\mathbf{y}) \longrightarrow$ the Hamming distance;
- $wt_H(\mathbf{x}) = d(\mathbf{x},\mathbf{0}) \longrightarrow$ the Hamming weight;
- q-ary code $\mathcal{C} \longrightarrow$ subset of $H(n,q)$;
- q-ary linear code $\mathcal{C} \longrightarrow$ linear subspace of $H(n,q)$;
- $d(\mathcal{C}) \longrightarrow$ the minimum distance of $\mathcal{C}$ $\min\{d(\mathbf{x},\mathbf{y}) \mid \mathbf{x},\mathbf{y} \in \mathcal{C},\ \mathbf{x} \neq \mathbf{y}\}$;
- $t(\mathcal{C}) \longrightarrow$ the packing radius of $\mathcal{C}$: $\left\lfloor \frac{d(\mathcal{C})-1}{2} \right\rfloor$;
- $\rho(\mathcal{C}) \longrightarrow$ the covering radius of $\mathcal{C}$: $\max_{\mathbf{x}\in H(n,q)} \min_{\mathbf{c}\in\mathcal{C}} d(\mathbf{x},\mathbf{c})$;
- A q-ary code $\mathcal{C}$ is called *quasi-perfect* if $\rho(\mathcal{C}) = t(\mathcal{C}) + 1$.

Open problems:

Problem 1: Are there any quasi-perfect codes of minimum distance 5 over the finite field $GF(q)$ for a prime power $q \geq 5$?

Problem 2: Are there any quasi-perfect codes BCH codes[1] over the finite field $GF(q)$ for a prime power $q \geq 5$?

Problem 3: Are there any quasi-perfect BCH codes of minimum distance at least 7?

Problem 4: Are there any quasi-perfect codes of minimum distance at least 9?

Problem 5: Is there an upper bound on the minimum distance of quasi-perfect codes?

References

[1] MacWilliams, F. J. and Sloane, N. J. A., *The Theory of Error-Correcting Codes*, North-Holland, Amsterdam (1977).

Author: Thomas Ericson
Title: Preparata codes over the GF(4)

Some definitions: The binary Preparata codes can be constructed with help of some code over $GF(4)$ and applying the standard Gray mapping from $GF(4)$ to $GF(2)^2$. Let $z : GF(2^\mu) \mapsto GF(4)$. We define Π_μ to be the set of all functions z with the following properties

1) $\sum_{u \in GF(2^\mu)} z(u) = 0$;
2) $\sum_{u \in GF(2^\mu)} u\mathrm{Tr}\{z(u)\} = 0$;
3) $\sum_{u \in GF(2^\mu)} u^3\mathrm{Tr}\{z(u)\} = \left(\sum_{u \in GF(2^\mu)} uz(u)\right)^3$;

Open problems:

Problem 1: Define an operation $\boxplus$ on Π_μ, i.e $\boxplus : \Pi_\mu^2 \mapsto \Pi_\mu$ such that Π_μ is an abelian group under this operation.

Problem 2: Find a similar $GF(4)$ description of the Kerdock codes.

Author: Denis Krotov
Title: On the binary codes with parameters of doubly-shortened 1-perfect codes

1. For q-ary alphabet $F_q = \{a_0, a_1, \ldots, a_{q-1}\}$, we define the map $\rho : F_q \to F_2^{q-1}$ by $\rho(a_0) = 00...00$, $\rho(a_1) = 10...00$, $\rho(a_2) = 01...00$, $\rho(a_{q-1}) = 00...01$. This map is coordinate-wise expanded to $\rho : F_q^n \to F_2^{n(q-1)}$.

By 2-rank of a code $C \subset F_q^n$ we mean the dimension of the affine span of $\rho(C)$.

2-Rank is invariant with respect to the isometries of the Hamming space (automorphisms of the Hamming graph).

Open problems:

Problem 1: Study the 2-rank of q-ary (perfect) codes (minimal, maximal values, ...; cases of odd and even q).

Problem 2: Find other "measures of non-linearity" that are invariant with respect to the isometries of the Hamming space.

2. By s-fold MDS code we mean a set of vertices of the Hamming graph H_q^n such that every line (maximal clique) contains exactly s code vertices. (1-Fold MDS codes are known as distance-2 MDS codes, Latin hypercubes, multary quasigroups.)

For $q \geq 4$, the number of such objects is known to be doubly-exponential (i.e., at least $2^{2^{cn}}$) [1].

By s-fold 1-perfect code we mean a set of vertices of the Hamming graph such that every ball of radius 1 contains exactly s code vertices.

Open problem:

Problem 3: Find the limit

$$\lim_{n\to\infty} \frac{\log\log(\text{the number of ...})}{n}$$

for s-fold MDS codes, s-fold 1-perfect codes. In particular, prove that this limit exists.

In some sense minimal unsolved case is the case of 2-fold MDS codes in H_4^n, while the 1-fold-MDS case in H_4^n can be solved by different approaches, see http://www.nsc.ru/ws/Lyap2001/2363/. Another important class is the class of $(n+1)/2$-fold 1-perfect binary codes, which exist for every odd n.

References

[1] D. S. Krotov, V. N. Potapov, P. V. Sokolova. On reconstructing reducible n-ary quasigroups and switching subquasigroups // Quasigroups and Related Systems 16(1) 2008, 55-67.

Author: Ivan Yu. Mogilnykh
Title: On perfect 2-colorings of Johnson graphs

Some necessary definitions:

Definition 1 (perfect coloring): Perfect m-coloring of a graph G with the matrix $A = \{a_{ij}\}_{i,j=1,\dots,m}$ is a coloring of the vertices of G into the set of colors $\{1,\dots,m\}$ such that the number of vertices of the color j adjacent with the fixed vertex x of the color i does not depend on a choice of the vertex x and equals to a_{ij}.
Matrix A is called a *matrix of parameters* of perfect coloring.

Such objects as 1-perfect constant weight code and $(w-1)-(n,w,\lambda)$-design can be defined as perfect 2-colorings of Johnson graph $J(n,w)$. The existence of 1-perfect constant weight codes is a long stating open question. The best result (no 1-perfect codes in Johnson graphs $J(n,w)$ exist for $n \leq 2^{250}$) is due to D.Gordon [4]

Open problems:

Problem 1(Delsarte [3]): Prove that no 1-perfect codes exist in Johnson graph.

Problem 2: The problem of existence of perfect 2-colorings: List all matrices of parameters of perfect 2-colorings of Johnson graphs $J(n,w)$ for small $n, n \geq 9$. For $n = 9$ two left open cases are colorings with matrices $\begin{pmatrix} 10 & 8 \\ 8 & 10 \end{pmatrix}$ for $J(9,3)$ and $\begin{pmatrix} 12 & 8 \\ 8 & 12 \end{pmatrix}$ for $J(9,4)$.

Problem 3:

Background: Using antipodality of 1-perfect codes, Avgustinovich in [1] established that 1-perfect code in Hamming space is uniquely defined by its "middle level" words – set of all codewords of weight $(n-1)/2$. More over, there are explicit formulas for reconstruction of perfect code by it's middle level (see [2]).

All perfect 2-colorings of Johnson graph $J(2w,w)$ hold the antipodality property simular to antipodality property of perfect codes in the Hamming space. For odd w this property implies that any perfect 2-coloring of graph $J(2w,w)$ is uniquely defined by it's 'middle level' – part of the coloring on vertices of graph that are at Johnson distance $w/2$ and $w/2+1$ from pair of two fixed antipodal vertices of $J(2w,w)$.

Problem statement: Find the formulas for reconstruction of perfect 2-colorings of J(2w,w) for odd w from the middle level of a graph.

References

[1] *S.V. Avgustinovich* On a property of perfect binary codes, Discrete Analysis and Operation Research (in Russian). 1995. V. 2, N. 1, P. 4-6.

[2] *S. V. Avgustinovich and A. Y. Vasil'eva* Testing Sets for 1-Perfect Codes, General Theory for Information Transfer and Combinatorics 2006. Lecture Notes in Comput. Sci. V. 4123. P. 938-940.

[3] *Delsarte P.* An Algebraic Approach to the Association Schemes of Coding Theory, Philips Res. Rep. Suppl. 1973. V. 10. P. 1-97.

[4] *Gordon M.D.* Perfect Single Error-Correcting Codes in the Johnson Scheme, IEEE Trans. Inform. Theory. 2006. V. 52, N 10. P. 4670-4672.

Authors: Patric Östergård and Olli Pottonen
Title: Classification and properties of the Perfect One-Error-Correcting Codes of Length 15

Open problem:

1. A binary code with length n, size M, and minimum distance at least d is an (n, M, d) code. The classification of binary one-perfect codes of length 15 in [2] leaves two undetermined cases for optimal binary one-error-correcting codes of length at most 15, namely the classification of $(12, 256, 3)$ and $(13, 512, 3)$ codes. An alternative path to the classification of $(15, 2048, 3)$ codes would give those classification results as well.

Open problems:

1) Is it possible to carry out a classification of binary one-perfect codes of length 15 via subcodes? Starting from $(8, 16, 3)$ codes, with repeated lengthening (clique search) and isomorph rejection.
2) Are there necessary properties of $(8, 16, 3)$ shortened codes that can be used to reject some candidates immediately?

2. In [3] two $(13, 512, 3)$ codes are discovered that are not subcodes of $(15, 2048, 3)$ codes. There are some interesting questions related to these two codes, listed in Table I.

TABLE I
TWO $(13, 512, 3)$ CODES

First code:			
Automorphism group generators:			
$(1\ 3\ 2\ 13)(\bar{4}\ \bar{7}\ \bar{8}\ 9)(5\ 10\ 6\ \overline{11})$		$(\bar{1}\ 3\ \bar{2}\ 13)(\bar{4}\ 8)(\bar{5})(\bar{6})(10\ \overline{11})(\overline{12})$	
$(3\ 13)(\bar{4}\ \bar{9})(5\ 10)(\bar{6}\ \overline{11})(\bar{7}\ \bar{8})(\overline{12})$		$(\bar{3}\ \overline{13})(4\ 10)(\bar{5}\ \bar{9})(\bar{6}\ \bar{7})(\bar{8}\ \overline{11})(\overline{12})$	
Orbit representatives:			
0000000000000	1000000010100	1000011001100	1010010000100
Second code:			
Automorphism group generators:			
$(\bar{3}\ 7)(\bar{4}\ \overline{13}\ \bar{6}\ \bar{8})(5\ \overline{11})(\bar{9})(\overline{10})(\overline{12})$		$(4\ 6)(\bar{5})(8\ 13)(\bar{9})(10\ 12)(\overline{11})$	
$(1\ \bar{7}\ 3)(\bar{2})(4\ \overline{13}\ \overline{10})(\bar{5}\ \bar{9}\ \overline{11})(6\ \bar{8}\ \overline{12})$			
Orbit representatives:			
0000000000000	1000000111000	1010100101000	
0000001101000	0010101111000	1000000001010	

Open problems:

1) Is there some nice explanation for the two codes?
2) Do the two codes have some property that directly implies that they cannot be lengthened to binary one-perfect codes of length 15?
3) Do the two codes belong to some (infinite) family of codes, which would lead to similar results for binary one-perfect codes of other lengths than 15.
4) Are there even more optimal binary one-error-correcting codes of length 13?

The last question is obviously related to the previous open problem.

3. There are some really challenging problems regarding the binary one-perfect codes of length 15, mentioned in [4] and stated in a more general form by Etzion and Vardy [3].

Open problems (see also the open problems of F.Solov'eva and M.Villanueva):

1) Determine the spectrum of intersection numbers of two codes, that is, all possible values of $|C_1 \cap C_2|$ when C_1 and C_2 are binary one-perfect codes of length 15.
2) Classify the partitions of $\mathbb{F}_2^{15}$ into one-perfect codes. Even the restricted version, for 16 equivalent codes in the partition, seems very hard.

Partial results for the spectrum of intersection numbers have been published in [3], and results for the partitioning problem include [9].

4. In [4] a wide variety of properties are studied for the binary one-perfect codes of length 15. The results form a good starting point for making conjectures about properties of longer binary one-perfect codes and trying to prove these. Here is just one example.

Open problems:

1) Find an example of a Steiner triple system of some order $2^n - 1$, $n \geq 5$ that does not occur in a binary one-perfect code of length $2^n - 1$.
2) Even better, show that such examples exist for all admissible orders.

REFERENCES

[1] Etzion, T. and Vardy, A. On perfect codes and tilings: Problems and solutions, *SIAM J. Discrete Math.* **11** (1998), 205–223.

[2] Östergård, P. R. J. and Pottonen, O., The perfect binary one-error-correcting codes of length 15: Part I—Classification, *IEEE Trans. Inform. Theory* **55** (2009), 4657–4660.

[3] Östergård, P. R. J. and Pottonen, O., Two optimal one-error-correcting codes of length 13 that are not doubly shortened perfect codes, *Des. Codes Cryptogr.*, to appear.

[4] Östergård, P. R. J., Pottonen, O., and Phelps, K. T., The perfect binary one-error-correcting codes of length 15: Part II—Properties, *IEEE Trans. Inform. Theory*, to appear.

[5] Solov'eva, F. I., On transitive partitions of an n-cube into codes (in Russian), *Problemy Peredachi Informatsii* **45**(1) (2009), 27–35. English translation in *Probl. Inf. Transm.* **45** (2009), 23–31.

Author: Fabio Pasticci
Title: Quasi-perfect linear codes with distance 4

1. Some necessary definitions:
Definition 1: An n-cap in a Galois space is a set of n points no three of which are collinear.
Definition 2: $A \subseteq AG(2,q)$ is a bi-covering cap in $AG(2,q)$ if each point $P \in AG(2,q)\setminus A$ is both an internal point to some segment of A, and an external point to some other segment of A.

Problem 1: Find new examples of bi-covering caps not contained in a conic or a cubic

2. Some necessary definitions:

Definition 3: An n-cap in a Galois space is a set of n points no three of which are collinear.

Some known results concerning the problem:
Theorem 1: There are positive constants c and M such that the following holds. In every projective plane of order $q \geq M$, there is a complete cap of size at most $\sqrt{q}\log^c q$

Problem 2: Find explicit constructions of caps attaining the bound by Kim and Vu.

References

[1] M. Giulietti, F. Pasticci, Quasi-perfect linear codes with minimum distance 4, IEEE Trans. Inform. Theory 53 (2007), no. 5, 1928–1935.
[2] J. H. Kim and V. H. Vu, Small complete arcs in projective planes, Combinatorica 23 (2003), no. 2, 311–363.

Author: Kevin T. Phelps
Title: An enumeration of Kerdock codes of length 64

1. Some necessary definitions:
Definition 2 (Kerdock-like code): A generalized Kerdock code or Kerdock-like code is a binary code of length 2^{m+1}, $m \geq 3$ and odd, and minimum distance $2^m - 2^{(m-1)/2}$ having weight distribution:

$weight$	$\#codewords$
0	1
$2^m - 2^{(m-1)/2}$	$2^{m+1}(2^m - 1)$
2^m	$2^{m+2} - 2$
$2^m + 2^{(m-1)/2}$	$2^{m+1}(2^m - 1)$
2^{m+1}	1

Definition 3 (Kerdock code): A binary code of length 2^{m+1}, $m \geq 3$ and odd, and minimum distance $2^m - 2^{(m-1)/2}$, consisting of the first order Reed-Muller code $RM(1, m+1)$ and $2^m - 1$ of its cosets having the above weight distribution.
Some known results concerning the problem: Every $\mathbb{Z}_4$-linear Kerdock-like code is a Kerdock code (Borges, Phelps, Rifa, Zinoviev)

Open problem:

Question: Are there Kerdock-like codes that are not Kerdock codes?

2. Some necessary definitions:
Graph $G(m)$, m odd

- Vertices: cosets of $RM(1, m+1)$ having minimum weight $2^m - 2^{(m-1)/2}$
- Edges: distance between cosets is $2^m - 2^{(m-1)/2}$
- Graph is multipartite with $2^m - 1$ parts.
- Kerdock set is clique of size $2^m - 1$

Some known results concerning the problem:

- $AGL(m+1)$ affine general linear group is automorphism group of $RM(1, m+1)$ (and $RM(2, m+1)$).
- Graph $G(m)$ is transitive and regular.

Open problem:

Problem 1: Find recurrence relation for vertex degree in $G(m)$.
Problem 2: Find the automorphism group of $G(m)$.
Problem 3: Find the smallest m such that there are at least 2 non-equivalent Kerdock-like codes of length 2^{m+1}.

REFERENCES

[1] J. Borges, K. P. Phelps, J. Rifà and V. Zinoviev, *On $\mathbb{Z}_4$-linear Preparata-like and Kerdock-like codes*, IEEE Transactions on Information Theory 49, n.11, pp. 2834-2843, 2003.

Author: Svetlana A. Puzinina
Title: Equitable partitions as a generalization of perfect codes

1. Let $G = (V, E)$ be a graph, $M = (m_{ij})_{i,j=1}^{n}$ an integer nonnegative matrix. Consider a partition Π of V with cells C_1, …, C_n. We call Π *equitable* if, for any pair of cells (C_i, C_j) and a vertex u in C_i, the number of vertices in C_j adjacent to u does not depend on the choice of u, but only on the pair (i, j), and is equal to m_{ij}. A matrix M is *G-admissible*, if there exists an equitable partition of G with this matrix.

Open problem:

Let T be a set of 2×2 nonnegative integer matrices, such that the sum of elements in each row is equal to m and nondiagonal elements are nonzero. The problem is to find a simple m-regular graph G, such that all matrices from T are G-admissible and there are no other G-admissible matrices of order two (if such graph exists).

2. Denote by $G(\mathbb{Z}^m)$ the graph of the infinite m-dimensional grid. The set of vertices of this graph consists of all m-tuples of integers. Two vertices are adjacent, if their m-tuples differ in one coordinate by unit.

Open problem:

Find the classification of $G(\mathbb{Z}^m)$-admissible matrices.

Remark. For $m \leq 3$ the solution is done.

References

[1] C. Godsil C. *Equitable partitions*. Combinatorics, Paul Erdős is Eighty Vol. 1. Budapest, 1993. p. 173–192.

Author: Faina I. Solov'eva
Titles: Perfect codes and related problems (introduction lecture);
Partitions of F_q^n into perfect codes

1. Kabatyanski and Panchenko in 1988, see [2], proved the following
Theorem. The density of the best parkings and coverings of $F_q^n, q \geq 2$ with the balls of radius $r = 1$ tends to 1 for $n \longrightarrow \infty$.

Two old brilliant challenging problems:

1) Find the density of the best parking and covering of A^n, $A = \{1, 2, \ldots, t\}$, with the balls of radius 1, where t is not a power of a prime.
2) Find the density of the best parking and covering of $F_q^n, q \geq 2$ with the balls of radius $r > 1$.

2. It is known the following
Theorem. The number $N(n)$ of nonisomorphic Steiner triple systems of order n satisfies the following bounds

$$(e^{-5}n)^{\frac{n^2}{6}} \leq N(n) \leq (e^{-1/2}n)^{\frac{n^2}{6}}.$$

The lower bound was proved by Egorychev in [1], 1980, using the result concerning permanents of double stochastic matrices, the upper bound is straightforward.

One more old brilliant challenging problem:

1) Improve the lower and upper bounds on the number of nonisomorphic Steiner triple systems presented in the previous theorem.

3. Two partitions of F_q^n into codes are called *different* if they differ in at least one code. Two partitions we call *equivalent* if there exists an isometry of the space F_q^n that transforms one partition into another one.

Open problems (see also open problems of P. Östergård and O. Pottonen, and M. Villanueva):

1) Find the classification of all partitions into perfect codes in $F_q^n, q \geq 2$.
2) Find the classification of all partitions into extended perfect codes in F_2^{16}.
3) Determine the spectrum of intersection numbers of any two q-ary perfect codes, i.e., all possible values of $|C \cap D|$ where C and D are q-ary perfect codes of length n.

Some contributions concerning the problems can be found in the papers mentioned in the list of references.

References

[1] *Egorychev G. P.,* The solution of the van der Waerden problem for permanents, Adv. in Math. (42) 3 (1981) 299–305.
[2] *Kabatyanskij G. A., Panchenko V. I.,* Packing and covering of the Hamming space by balls of unit radius, Problems of information Transm. (4) 24 (1988) 3–16
[3] Etzion, T. and Vardy, A. On perfect codes and tilings: Problems and solutions, *SIAM J. Discrete Math.* **11** (1998), 205–223.
[4] *F. I. Solov'eva* "On perfect codes and related topics," Com^{2}Mac Lecture Note. Series 13. Pohang, 2004. 80 p.

[5] S. V. Avgustinovich, O. Heden, F. I. Solov'eva, "On intersections of perfect binary codes", *Bayreuther Mathematische Schriften*, vol. 71, pp. 8–13, 2005.
[6] S. V. Avgustinovich, O. Heden, F. I. Solov'eva, "On intersection problem for perfect binary codes", *Des. Codes Crypt.*, vol. 39, pp. 317–322, 2006.
[7] E. Bar-Yahalom, T. Etzion, "Intersection of isomorphic linear codes", *Journal of Comb. Theory*, Series A 80, pp. 247–256, 1997.
[8] *F. I. Solov'eva, A. V. Los'* "Constructions of partitions of F_q^n into perfect q-ary codes," Discrete Analysis and Oper. Research, **16** (3) (2009), 63–73 (in Russian).
[9] Solov'eva, F. I., On transitive partitions of an n-cube into codes (in Russian), *Problemy Peredachi Informatsii* **45**(1) (2009), 27–35. English translation in *Probl. Inf. Transm.* **45** (2009), 23–31.

Author: Mercè Villanueva

Title: $\mathbb{Z}_2\mathbb{Z}_4$-additive (extended) perfect codes: intersection problem

1. Open problem (see also open problems of P. Östergård and O. Pottonen, and F. Solov'eva):

For a given t, find the possible intersection numbers of distinct binary perfect codes of length $n = 2^t - 1$. In general, for a given q and t, find the possible intersection numbers of distinct q-ary perfect codes of length $n = \frac{q^t-1}{q-1}$.

2. For two binary codes C_1, C_2, define $i(C_1, C_2) = |C_1 \cap C_2|$ to be their intersection number. A *Hadamard matrix* H of order n is an $n \times n$ matrix of $+1$'s and -1's such that $HH^T = nI$, where I is the $n \times n$ identity matrix. If $+1$'s are replaced by 0's and -1's by 1's, H is changed into a *binary Hadamard matrix* $c(H)$. The binary $(n, 2n, n/2)$-code consisting of the rows of $c(H)$ and their complements is called a *(binary) Hadamard code*. It is known that there exist Hadamard codes of length 2^t, for all $t \geq 3$, with intersection number i if and only if $i \in \{0, 2, 4, \ldots, 2^{t+1} - 12, 2^{t+1} - 8, 2^{t+1}\}$ [2]. Moreover, for all $t \geq 4$, if there exists a Hadamard matrix of order $4s$, then there exist Hadamard codes of length $2^{t+2}s$ with intersection number i if and only if $i \in \{0, 2, 4, \ldots, 2^{t+3}s - 12, 2^{t+3}s - 8, 2^{t+3}s\}$ [2].

Open problem: Find the possible intersection numbers of distinct Hadamard codes of length $4s$, for $s > 1$, s odd.

3. There are new constructions to obtain families of $\mathbb{Z}_2\mathbb{Z}_4$-additive codes such that, under the Gray map, the corresponding binary codes have the same parameters and properties as the usual binary linear Reed-Muller codes [3], [4]. These families include the $\mathbb{Z}_2\mathbb{Z}_4$-additive extended perfectes codes and $\mathbb{Z}_2\mathbb{Z}_4$-additive Hadamard codes. It is known a complete solution for the intersection problem for $\mathbb{Z}_2\mathbb{Z}_4$-additive Hadamard codes and $\mathbb{Z}_2\mathbb{Z}_4$-additive extended perfect codes [5], [6].

Open problem:

Study the classification of these new families of codes. Give a complete solution for the intersection problem of these new families of $\mathbb{Z}_2\mathbb{Z}_4$-additive Reed-Muller codes.

References

[1] MacWilliams, F. J. and Sloane, N. J. A., *The Theory of Error-Correcting Codes*, North-Holland, Amsterdam (1977).

[2] Phelps K. T., Villanueva M., Intersection of Hadamard Codes, *IEEE Transactions on Information Theory*, (2007) 53, n. 5: 1924-1928.

[3] Pujol J., Rifà J., and Solov'eva F. I., Construction of $\mathbb{Z}_4$-Linear Reed-Muller Codes *IEEE Transactions on Information Theory* (2009) 55, n. 1: 99-104.

[4] Pujol J., Rifà J., and Ronquillo L., Construction of Additive Reed-Muller Codes, *Applicable Algebra in Engineering, Communication and Computing* (2009) 5527: 223-226.

[5] J. Rifà, F. I. Solov'eva and M. Villanueva, "On the intersection of $\mathbb{Z}_2\mathbb{Z}_4$-additive perfect codes", *IEEE Trans. Inform. Theory*, vol. 54, no. 3, pp. 1346–1356, 2008.

[6] J. Rifà, F. I. Solov'eva and M. Villanueva, "On the intersection of $\mathbb{Z}_2\mathbb{Z}_4$-additive Hadamard codes", *IEEE Trans. Inform. Theory*, vol. 55, no. 4, pp. 1766–1774, 2009.

Author: Thomas Westerbäck
Title: On the existence of perfect and extended perfect binary codes with trivial symmetry group

Open problems:

Problem 1: What can be said about the existence of perfect codes of length $n = 2^m - 1$ and rank $n - m + 2$ with a trivial symmetry group?

Problem 2: What can be said in general about the symmetry groups of perfect and extended perfect codes with different rank and dimensions of the kernel?

Author: Victor A. Zinoviev
Title: On Preparata-like codes and 2-resolvable Steiner quadruple systems

Open problems:

Problem 1: Whether any Preparata-like code P of length n induces a partition of the corresponding Hamming-like code of length n into disjoint Preparata-like codes?

Problem 2: Are there other cases of Hamming-like codes H (different from the (linear) Hamming code and Z_4-linear Hamming-like code), which contain some Preparata-like code P?