MODEL SET VALIDATION AND UPDATE FOR TIME-VARYING SISO SYSTEMS

Ryozo Nagamune* **and Shigeru Yamamoto****

* *Division of Optimization and Systems Theory, Royal Institute of Technology, SE 100 44 Stockholm, Sweden, Email: ryozo@math.kth.se*

** *Department of Systems and Human Science, Graduate School of Engineering Science, Osaka University, 1-3 Machikaneyama, Toyonaka, Osaka 560-8531, Japan, Email: yamamoto@sys.es.osaka-u.ac.jp*

Abstract. This paper presents a method of the model set validation (VD) and update (UD) for time-varying SISO systems by using time-domain data. As a representation of such systems, we use a discrete-time linear difference equation with time-varying parametric uncertainties measured by the weighted ∞-norm. It is shown that the validation can be simply conducted by comparisons of distances in a parameter space, and that the update is done by solving a convex optimization problem in general, and by giving an explicit solution in a special case. Based on these results, we propose an off- and on-line algorithm of the model set VD and UD, respectively.

Key Words. model set validation and update, time-varying systems, parametric uncertainties, convex optimization

1. Introduction

Robust control theory requires a model set which reflects characteristics of a real system. To construct a model set with high reliability is essential for robust control. However, it is hard to obtain the model set which catches all features of the real system due to various uncertainties caused by the finiteness of the data, the effect of disturbances and noise on the data for identification, the high complexity of the real system to be modeled, etc. Therefore, it is important to test the validity of the model set obtained by some identification method. We call this stage 'model set validation.' In addition, if the model set is invalidated by the model set validation, it should be updated into a model set with much reliability. We refer to this stage as 'model set update.'

Model set validation has been argued in many papers so far (see [1], [2], [3]). In spite of these contributions, little attention has been given to a validation problem for a model set with time-varying parametric uncertainties. Moreover, model set update problems have not fully been commented on in those papers and are still open.

The aim of this paper is to propose a method of a series of model set VD and UD for time-varying SISO systems, by using time domain data. The model set treated in this paper is a discrete-time linear difference equation with time-varying parametric uncertainties measured by the weighted ∞-norm. For this model set, we will formulate model set VD and UD problems. It is shown that the VD problem is solved via simple calculations and comparisons of distances in a parameter space, while the UD problem can be reduced to a convex optimization problem in general. Furthermore, it is also shown that the UD problem in a special case has an explicit solution. Based on these results, we shall propose an algorithm of model set VD and UD in an off- and on-line case, respectively.

We have already proposed an identification (ID) method of the model set described by the linear difference equation with time-varying parameters in [4]. It is expected that the proposed model set VD and UD algorithms are useful for increasing reliability of a model set obtained by the ID method.

Notations used in this paper are standard. For a vector v, $\|v\|_\infty$ and $\|v\|_1$ denote the ∞- and the 1-norm respectively, and are defined by $\|v\|_\infty := \max_i |v_i|$ and $\|v\|_1 := \sum_i |v_i|$. For a matrix M, M^T denotes the transpose of M. For a real number c, $\mathrm{sgn}(c)$ is the signum function of c, defined by

$$\mathrm{sgn}(c) := \begin{cases} 1, & c > 0, \\ 0, & c = 0, \\ -1, & c < 0. \end{cases}$$

Similarly, for a vector $v = [v_1, \cdots, v_n]^T$,

$$\mathrm{sgn}(v) := [\mathrm{sgn}(v_1), \cdots, \mathrm{sgn}(v_n)]^T .$$

2. Problem Formulation

Assume that we are given the following model set whose

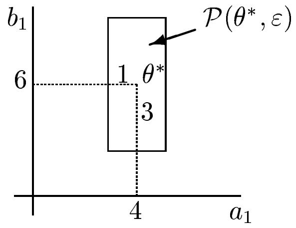


FIGURE 1: $\mathcal{P}(\theta^*, \varepsilon)$ ($\theta^* = [4, 6]^T, \varepsilon = 3, W = \mathrm{diag}[3, 1]$)

validity is tested from now on:

$$\mathcal{M} = \left.\begin{array}{lll} \{y(t) = \varphi^T(t)\theta(t), & & \\ \quad \theta(t) & := & [a_1, \cdots, a_n, b_1, \cdots, b_n]^T \\ & \in & \mathcal{P}(\theta^*, \varepsilon) \end{array}\right\} . \tag{1}$$

In (1), $\varphi(t)$ is a regression vector defined by

$$\varphi(t) := [-y(t-1), \cdots, -y(t-n), \\ u(t-1), \cdots, u(t-n)]^T ,$$

where $u(t)$ and $y(t)$ are, respectively, observed values of the input and output at time t, and n denotes a known order of a real system. Time-varying parameters a_i and b_i are unknown but assumed that a parameter vector $\theta(t)$ belongs to a set $\mathcal{P}(\theta^*, \varepsilon)$:

$$\mathcal{P}(\theta^*, \varepsilon) := \{\theta \mid d(\theta^*, \theta) \le \varepsilon\} , \tag{2}$$

where a vector $\theta^* := [a_1^*, \cdots, a_n^*, b_1^*, \cdots, b_n^*]^T$ is a center of the set $\mathcal{P}(\theta^*, \varepsilon)$ and a scalar ε is a bound of parametric uncertainties and $d(\theta^*, \theta)$ is a distance between θ^* and θ. As a measure of the distance $d(\theta^*, \theta)$, we will use the following weighted ∞-norm:

$$d(\theta^*, \theta) := \|W(\theta - \theta^*)\|_\infty .$$

Here, $W := \mathrm{diag}\,[w_1, \cdots, w_{2n}] > 0$ denotes a given fixed weighting matrix and determines a shape of the parameter set $\mathcal{P}(\theta^*, \varepsilon)$. Figure 1 depicts a simple example of $\mathcal{P}(\theta^*, \varepsilon)$.

Remark 1 This type of model sets are assumed to be obtained by an appropriate model set ID method. Such a model set ID problem has been considered in [4].

In the sequel, we will formulate model set VD and UD problems for a given model set (1). As *a priori* information on a real system, assume that

- the initial conditions of the system are zero,
- the order of the real system is n,
- the real system is strictly causal and
- the input $u(t) \equiv 0$ for time $t < 0$.

We also assume that we are given scalar noise-free input/output measurements which has been recorded at $N + 1$ integer sample times $k = 0, \cdots, N$, that is,

$$\{u(k), y(k)\} , \quad k = 0, \cdots, N. \tag{3}$$

Our objective is to determine the validity of the model set $\mathcal{M}$ under a given experimental data (3). If the data (3) cannot be generated by an element in $\mathcal{M}$, then we should update $\mathcal{M}$ in some ways. In this paper, we consider an updating method of changing the center θ^* and the bound ε of $\mathcal{P}(\theta^*, \varepsilon)$. Now, we assume that the model set $\mathcal{M}$ has been constructed by using insufficient number of data, and that we should test the validity because we would like to complement the shortage of the data. In such a situation, a new parameter set $\mathcal{P}(\theta^*_{\mathrm{new}}, \varepsilon_{\mathrm{new}})$ must contain the initial parameter set $\mathcal{P}(\theta^*, \varepsilon)$, that is,

$$\mathcal{P}(\theta^*, \varepsilon) \subseteq \mathcal{P}(\theta^*_{\mathrm{new}}, \varepsilon_{\mathrm{new}}),$$

in order to guarantee the consistency of $\mathcal{P}(\theta^*_{\mathrm{new}}, \varepsilon_{\mathrm{new}})$ with the data used in construction of $\mathcal{M}$.

We now formulate the model set validation and update problems considered in this paper.

Problem *(Model set VD and UD) Assume that we are given both*

- *a model set $\mathcal{M}$ to be invalidated (that is, θ^* and ε) and*
- *data (3) from a real system for VD.*

The model set VD problem is to determine whether there exists at least one model in the model set $\mathcal{M}$ which can produce the given data (3) or not. If the answer to this problem is negative, it is necessary to update the model set $\mathcal{M}$. The model set UD problem is to find a new model set $\mathcal{M}_{\mathrm{new}}$:

$$\mathcal{M}_{\mathrm{new}} = \left\{y(t) = \varphi^T(t)\theta(t) \mid \theta(t) \in \mathcal{P}(\theta^*_{\mathrm{new}}, \varepsilon_{\mathrm{new}})\right\} ,$$

which has the following properties:

- *$\mathcal{M}_{\mathrm{new}}$ produces the given data (3),*
- *$\mathcal{P}(\theta^*_{\mathrm{new}}, \varepsilon_{\mathrm{new}})$ contains $\mathcal{P}(\theta^*, \varepsilon)$ and*
- *$\mathcal{M}_{\mathrm{new}}$ ($\varepsilon_{\mathrm{new}}$) is minimized.*

Remark 2 The reason for adding the last condition that 'a new model set is minimized' is that it is preferable for the robust controller design to construct a model set *as small as possible*.

This problem is reformulated mathematically as follows. Substitution of the data (3) into $y(t) = \varphi^T(t)\theta(t)$ yields the following parameter set $\mathcal{D}_k$:

$$\mathcal{D}_k := \left\{\theta(k) \mid y(k) = \varphi^T(k)\theta(k)\right\} , \ k = 1, \cdots, N.$$

$$(y(k) \equiv 0 \text{ for } k \le 0 \text{ and } u(k) \equiv 0 \text{ for } k < 0.)$$

$\mathcal{D}_k$ is a parameter set which can produce a data pair $\{y(k), \varphi(k)\}$ at time k. Consider the $2n$ dimensional Euclidean space $\boldsymbol{R}^{2n}$. Then $\mathcal{D}_k$ forms a hyperplane and $\mathcal{P}(\theta^*, \varepsilon)$ in (2) forms a hyper-rectangle in $\boldsymbol{R}^{2n}$. Here, we can rewrite the problem in terms of parameter sets $\mathcal{P}(\theta^*, \varepsilon)$ and $\mathcal{D}_k$ as follows: Assume that we are given in the space $\boldsymbol{R}^{2n}$

- a hyper-rectangle $\mathcal{P}(\theta^*, \varepsilon)$ and
- hyperplanes $\mathcal{D}_k,\ k = 1, \cdots, N$.

The model set VD problem is to determine whether or not the relationship

$$\mathcal{P}(\theta^*, \varepsilon) \cap \mathcal{D}_k \neq \emptyset \tag{4}$$

holds for all $k = 1, \cdots, N$. If there exists at least one k at which (4) does not hold, the update problem is to find a new hyper-rectangle $\mathcal{P}(\theta^*_{\text{new}}, \varepsilon_{\text{new}})$:

$$\mathcal{P}(\theta^*_{\text{new}}, \varepsilon_{\text{new}}) = \{\theta \mid d(\theta^*_{\text{new}}, \theta) \leq \varepsilon_{\text{new}}\}$$

which has the following properties:

- $\mathcal{P}(\theta^*_{\text{new}}, \varepsilon_{\text{new}}) \cap \mathcal{D}_k \neq \emptyset$,
- $\mathcal{P}(\theta^*, \varepsilon) \subseteq \mathcal{P}(\theta^*_{\text{new}}, \varepsilon_{\text{new}})$ and
- ε_{new} is minimized.

3. Model Set Validation

The model set VD problem is to determine if the next relationship holds or not.

$$\mathcal{P}(\theta^*, \varepsilon) \cap \mathcal{D}_k \neq \emptyset, \quad k = 1, \cdots, N.$$

This condition is equivalent to for $k = 1, \cdots, N$,

$$d(\theta^*, \mathcal{D}_k) := \min_{\theta \in \mathcal{D}_k} \|W(\theta - \theta^*)\|_\infty \leq \varepsilon,$$

where $d(\theta^*, \mathcal{D}_k)$ is the distance from a point θ^* to a hyperplane $\mathcal{D}_k$ in the space $\boldsymbol{R}^{2n}$. Due to the result in [4], we can calculate the distance $d(\theta^*, \mathcal{D}_k)$ simply by

$$d(\theta^*, \mathcal{D}_k) = \frac{|y(k) - \varphi^T(k)\theta^*|}{\|W^{-1}\varphi(k)\|_1}.$$

Eventually, the model set validation test can be reduced to the comparisons of the distances $d(\theta^*, \mathcal{D}_k)$ with the value ε. If the inequality $d(\theta^*, \mathcal{D}_k) \leq \varepsilon$ holds for all $k = 1, \cdots, N$, then the parameter set $\mathcal{P}(\theta^*, \varepsilon)$ is not invalidated, and therefore, is not necessary to be updated. Otherwise, $\mathcal{P}(\theta^*, \varepsilon)$ is not valid, and $\mathcal{P}(\theta^*, \varepsilon)$ should be updated to a new set $\mathcal{P}(\theta^*_{\text{new}}, \varepsilon_{\text{new}})$ as is explained in the next section.

4. Model Set Update

We need to update the model set if there exists a hyperplane $\mathcal{D}_k$ such that $d(\theta^*, \mathcal{D}_k) > \varepsilon$. The model set UD problem is to find a new minimal parameter set $\mathcal{P}(\theta^*_{\text{new}}, \varepsilon_{\text{new}})$ meeting $\mathcal{P}(\theta^*_{\text{new}}, \varepsilon_{\text{new}}) \cap \mathcal{D}_k \neq \emptyset$ and $\mathcal{P}(\theta^*, \varepsilon) \subseteq \mathcal{P}(\theta^*_{\text{new}}, \varepsilon_{\text{new}})$. These conditions are equivalent to the conditions:

$$\begin{cases} \displaystyle\max_{1 \leq k \leq N} d(\theta^*_{\text{new}}, \mathcal{D}_k) \leq \varepsilon_{\text{new}}, \\ \displaystyle\max_{\theta \in \mathcal{P}(\theta^*, \varepsilon)} d(\theta^*_{\text{new}}, \theta) \leq \varepsilon_{\text{new}}, \end{cases}$$

where $\max_{\theta \in \mathcal{P}(\theta^*, \varepsilon)} d(\theta^*_{\text{new}}, \theta)$ means the maximal distance between θ^*_{new} and any point θ in $\mathcal{P}(\theta^*, \varepsilon)$. Hence, adding the minimal condition of ε_{new} to these conditions, we can reduce the model set UD problem to the following min-max problem:

$$\begin{aligned} \varepsilon_{\text{new}} = \min_{\theta^*_{\text{new}}} \max \Bigg\{ & \max_{1 \leq k \leq N} d(\theta^*_{\text{new}}, \mathcal{D}_k), \\ & \max_{\theta \in \mathcal{P}(\theta^*, \varepsilon)} d(\theta^*_{\text{new}}, \theta) \Bigg\}. \end{aligned} \tag{5}$$

The next lemma shows that the maximization in the braces of (5) can be done in the finite number of calculations.

Lemma 1 *Solutions of the equation (5) are equivalent to solutions of the next equation:*

$$\begin{aligned} \varepsilon_{\text{new}} = \min_{\theta^*_{\text{new}}} \max \Bigg\{ & \max_{k \in \mathcal{I}} d(\theta^*_{\text{new}}, \mathcal{D}_k), \\ & \max_{\theta_i \in \mathcal{V}(\mathcal{P}(\theta^*, \varepsilon))} d(\theta^*_{\text{new}}, \theta_i) \Bigg\}, \end{aligned} \tag{6}$$

where a set $\mathcal{I}$ consists of all indices k at which $d(\theta^*, \mathcal{D}_k) > \varepsilon$ satisfies, and a set $\mathcal{V}(\mathcal{P}(\theta^*, \varepsilon))$ consists of all vertices θ_i of a hyper-rectangle $\mathcal{P}(\theta^*, \varepsilon)$. To be more specific,

$$\mathcal{I} := \{k \mid d(\theta^*, \mathcal{D}_k) > \varepsilon, k = 1, \cdots, N\}, \tag{7}$$

$$\left.\begin{aligned} \mathcal{V}(\mathcal{P}(\theta^*, \varepsilon)) := \{ & \theta_i := \theta^* + \varepsilon W^{-1}\delta_i, \\ & \delta_i = [\underbrace{\pm 1, \cdots, \pm 1}_{2n}]^T, i = 1, \cdots, 2^{2n} \end{aligned}\right\}, \tag{8}$$

where $\pm$ in (8) means all combinations.

The next theorem is a main result of this section, which guarantees the computational feasibility of the formulated update problem.

Theorem *The function in (6):*

$$\max \left\{ \max_{k \in \mathcal{I}} d(\theta^*_{\text{new}}, \mathcal{D}_k), \max_{\theta_i \in \mathcal{V}(\mathcal{P}(\theta^*, \varepsilon))} d(\theta^*_{\text{new}}, \theta_i) \right\}$$

is convex with respect to θ^*_{new}.

Due to this theorem, we can solve the equation (6) via a convex optimization, and obtain numerically the optimal solutions of θ^*_{new} and ε_{new} (Fig. 2).

Particularly, when $\mathcal{I}$ in (7) is a singleton, the computation becomes considerably simple (in fact, it has an explicit solution of θ^*_{new} and ε_{new}.) This fact is useful for the construction of an on-line algorithm of the model set VD and UD, as is described in the next section.

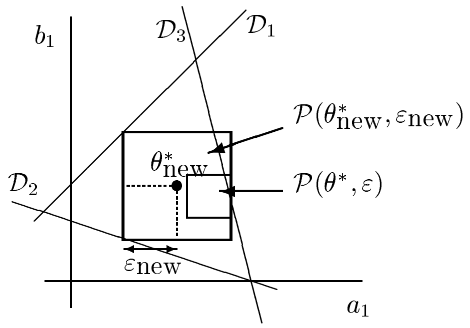


FIGURE 2: Validation and update of a set $\mathcal{P}(\theta^*, \varepsilon)$ (inner square) in the case where $n = 1$ and $W = I$: An updated hyper-rectangle $\mathcal{P}(\theta^*_{\text{new}}, \varepsilon_{\text{new}})$ (outer square) and hyperplanes $\mathcal{D}_k$ (straight lines)

Lemma 2 *If the set $\mathcal{I}$ has only one element, that is $\mathcal{I} = \{p\}$, then solutions to the model set UD problem:*

$$\varepsilon_{\text{new}} = \min_{\theta^*_{\text{new}}} \max \Big\{ d(\theta^*_{\text{new}}, \mathcal{D}_p), \max_{\theta_i \in \mathcal{V}(\mathcal{P}(\theta^*, \varepsilon))} d(\theta^*_{\text{new}}, \theta_i) \Big\}$$

can be given explicitly by

$$\theta^*_{\text{new}} = \theta^* + (d_p - \varepsilon) c W^{-1} v/2,$$

$$\varepsilon_{\text{new}} = (d_p + \varepsilon)/2.$$

where $d_p := d(\theta^*, \mathcal{D}_p) (> \varepsilon)$ is the distance between θ^* and $\mathcal{D}_p$, $c := \operatorname{sgn}(y(p) - \varphi^T(p)\theta^*)$ and $v := \operatorname{sgn}(\varphi(p))$.

Remark 3 Though the proof is omitted because of constraint of space, its geometric idea is depicted in Fig. 3.

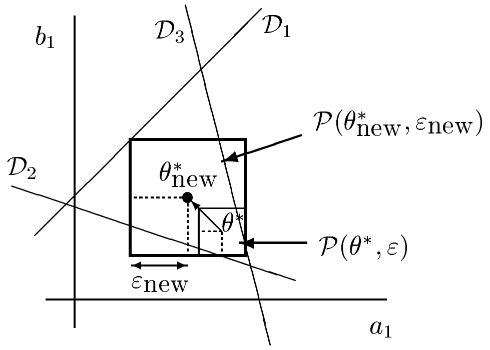


FIGURE 3: Geometric idea of Lemma 2 in the case where $n = 1$ and $W = I$: Note that $\mathcal{P}(\theta^*, \varepsilon) \cap \mathcal{D}_k \neq \emptyset$ does not hold for only $k = 1$, that is, $\mathcal{I} = \{1\}$ ($\mathcal{I}$ has only one element)

5. Validation-Update Algorithms

To summarize the results so far, we shall propose model set VD and UD algorithms as follows.

5.1. An Off-line Algorithm

Assume that we are given a parameter set

$$\mathcal{P}(\theta^*, \varepsilon) = \{\theta \mid d(\theta^*, \theta) \leq \varepsilon\}$$

to be investigated via model set validation.

Step 1. By using given data (3), calculate the distances $d(\theta^*, \mathcal{D}_k)$ as

$$d(\theta^*, \mathcal{D}_k) = \frac{|y(k) - \varphi^T(k)\theta^*|}{\|W^{-1}\varphi(k)\|_1}, \quad k = 1, \cdots, N.$$

Step 2. If the inequality $\max_{1 \leq k \leq N} d(\theta^*, \mathcal{D}_k) \leq \varepsilon$ holds, then $\theta^*_{\text{new}} = \theta^*$ and $\varepsilon_{\text{new}} = \varepsilon$. Otherwise, go to Step 3.

Step 3. θ^*_{new} and ε_{new} are obtained by solving a convex optimization problem (6).

5.2. An On-line Algorithm

A center and a bound of a parameter set which has not been invalidated until time k are assumed to be given as $\theta^*(k)$ and $\varepsilon(k)$, respectively.

Step 1. At time $(k+1)$, using data $\{y(k+1), \varphi(k+1)\}$, calculate $d(\theta^*(k), \mathcal{D}_{k+1})$ as

$$d(\theta^*(k), \mathcal{D}_{k+1}) = \frac{|y(k+1) - \varphi^T(k+1)\theta^*(k)|}{\|W^{-1}\varphi(k+1)\|_1}.$$

Step 2. If the inequality $d(\theta^*(k), \mathcal{D}_{k+1}) \leq \varepsilon(k)$ holds, then $\theta^*(k+1) = \theta^*(k)$ and $\varepsilon(k+1) = \varepsilon(k)$. Otherwise, go to Step 3.

Step 3. Defining $d_{k+1} := d(\theta^*(k), \mathcal{D}_{k+1}) (> \varepsilon(k))$, the values of $\theta^*(k)$ and $\varepsilon(k)$ are updated as

$$\theta^*(k+1) = \theta^*(k) + (d_{k+1} - \varepsilon(k)) c(k) W^{-1} v(k)/2,$$

$$\varepsilon(k+1) = (d_{k+1} + \varepsilon(k))/2,$$

where $c(k) := \operatorname{sgn}(y(k+1) - \varphi^T(k+1)\theta^*(k))$ and $v(k) := \operatorname{sgn}(\varphi(k+1))$.

6. A Numerical Example

We shall give a numerical example which was simulated according to the algorithms proposed in the previous section. As a real system, we assumed the following 1st-order time-varying system:

$$y(t) = -a(t)y(t-1) + b(t)u(t-1),$$

where time-varying parameters $a(t)$ and $b(t)$ were assumed to vary as follows:

$$\begin{aligned} a(t) &= -0.3 - 0.5\sin(0.3t), \\ b(t) &= 1.0 + 0.5\cos(0.15t). \end{aligned}$$

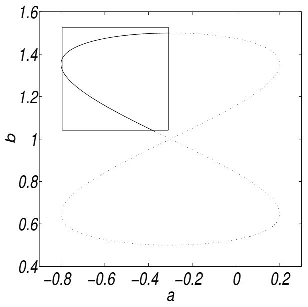


FIGURE 4: The trajectory of time-varying parameters $a(t)$ and $b(t)$ during the experiment for ID (the solid curve) and the obtained parameter set (the square)

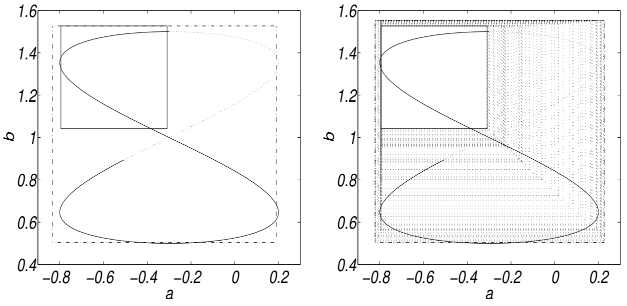
To construct a model set describing this system by the method proposed in [4], we sampled output for the white noise input for 10 seconds in the 0.01 second sampling period. When the weighting matrix $W = I$, the obtained parameter set was in the following (see Fig. 4):

$$\theta^* = [-0.5526, 1.2835]^T\ ,\ \varepsilon = 0.2422.$$

Next, to test the validity of the model set, we recorded 30 seconds of data in the same setting. The updated model sets were calculated in off- and on-line cases as follows (See Figure 5).

$$\begin{aligned} \text{Off-line case:}\quad & \theta^*_{\text{new}} = [-0.3212, 1.0152]^T, \\ & \varepsilon_{\text{new}} = 0.5106, \\ \text{On-line case:}\quad & \theta^*_{\text{new}} = [-0.2892, 1.0284]^T, \\ & \varepsilon_{\text{new}} = 0.5240. \end{aligned}$$

From the obtained values and Figure 5, we can see that the resulting model sets in off- and on-line cases have



(a) Off-line update

(b) On-line update

FIGURE 5: Update of the parameter set: the trajectory of time-varying parameters $a(t)$ and $b(t)$ during the experiment for validation (the solid curve), the tested parameter set (the smallest square) and the obtained parameter set (the largest square) via update

become close to the real system, which is desirable for model sets to be used in the robust controller design. These results imply that the proposed model set validation and update algorithms are useful for supplementing the number of the data used for identification of $\mathcal{M}$ and for increasing the reliability of the model set.

7. Conclusions

In this paper, we have formulated and solved model set validation and update problems for time-varying SISO systems. The model set treated in this paper was a discrete-time linear difference equation with time-varying parametric uncertainties measured by the weighted ∞-norm. The model set validation has become simple computations and comparisons of distances in a parameter space. Moreover, it was shown that the formulated model set update problem can generally be solved via a convex optimization, while it has an explicit solution in a special case. We used these results for constructing the model set validation and update algorithms in both off-line and on-line cases. We tested the validity of the proposed algorithms through a numerical example.

We have to consider the model set validation and update problems for model sets described by various other kinds of model structures, for example, time-varying state-space model sets or model sets in the LFT form. Additionally, the proposed methods in this paper should be extended to those considering noisy cases for practical use. Moreover, the proposed algorithms should be extended to those for MIMO systems. Furthermore, a method to determine a weighting matrix W should be clarified. These issues are still left for future research.

References

[1] R. S. Smith and J. C. Doyle, "Model validation: A Connection Between Robust Control and Identification," *IEEE Trans. Automat. Control*, vol. 37, no. 7, pp. 942–952, 1992.

[2] K. Poolla, P. P. Khargonekar, A. Tikku, J. Krause, and K. M. Nagpal, "A Time-domain Approach to Model Validation," *IEEE Trans. Automat. Control*, vol. 39, no. 5, pp. 951–959, 1994.

[3] J. Chen and S. Wang, "Validation of linear fractional uncertain models: Solutions via matrix inequalities," *IEEE Trans. Automat. Control*, vol. 41, no. 6, pp. 844–849, 1996.

[4] R. Nagamune, S. Yamamoto, and H. Kimura, "Identification of the smallest unfalsified model set with both parametric and unstructured uncertainty," in *Preprints of the 11th IFAC Symposium on System Identification (SYSID'97)*, vol. 1, (Kitakyushu, Japan), pp. 75–80, July 1997.