# Extremal Combinatorics examples sheet 3

*There are also exercises in the notes; some of these are included below and some are not, but you should attempt all exercises to ensure a thorough understanding of the course material. The examples sheets are unassessed, but you are welcome to hand in your attempts for feedback.*

1. Show that the Local LYM inequality follows from the LYM inequality.

2. Conversely (and more interestingly), deduce the LYM inequality from the Local LYM inequality. (*Hint: work your way down from the top layer.*)

3. Show that if equality holds in the Local LYM inequality then $\mathcal{A}$ is either empty or the complete layer $\binom{X}{r}$.

4. When does equality hold in the LYM inequality? (*Hint: the previous exercises can help with this!*)

5. Show that if equality holds in Sperner's theorem then the set system is a middle layer, that is $\mathcal{A} = \binom{X}{k}$ for $k = \lfloor n/2 \rfloor$ or $k = \lceil n/2 \rceil$. (The case when $n$ is even can be deduced somewhat more easily than the case when $n$ is odd.)

6. Write out $\binom{[5]}{3}$ ordered by colex. How does this relate to $\binom{[6]}{3}$ ordered by colex?

7. Let $r \in \mathbb{N}$. Show that any positive integer $m$ can be written uniquely in the form
$$m = \binom{m_r}{r} + \binom{m_{r-1}}{r-1} + \cdots + \binom{m_s}{s}$$
where $m_r > m_{r-1} > \cdots > m_s > 0$ and $r \geqslant s \geqslant 1$. (*Hint: think greedily.*)

8. Show that if $\mathcal{C}$ is an initial segment of colex of size
$$|\mathcal{C}| = \binom{m_r}{r} + \binom{m_{r-1}}{r-1} + \cdots + \binom{m_s}{s}$$
as above then
$$|\partial \mathcal{C}| = \binom{m_r}{r-1} + \binom{m_{r-1}}{r-1} + \cdots + \binom{m_s}{s-1}.$$

9. Show that an initial segment of colex is left-compressed.

Please let me know if you have any comments or corrections.

*E-mail address*: `O.Sisask@qmul.ac.uk`