EXERCISE SET II

Exercise 1. *Let R be a commutative ring with 1, and let $\mathfrak{m}_1, \ldots, \mathfrak{m}_n$ be distinct maximal ideals in R. Show that, for any positive integers $p_1, \ldots, p_n$ we have a natural isomorphism*

$$R/\mathfrak{m}^{p_1} \cdots \mathfrak{m}_n^{p_n} \longrightarrow \prod_{i=1}^{n} R/\mathfrak{m}_i^{p_i}.$$

Exercise 2. *Let $S \subseteq R$ be a multiplicatively closed subset of a commutative ring R, and assueme $1 \in S$. Show the universal property for the morphism $\iota : R \to S^{-1}R$. That is, let $\varphi : R \to R'$ be ring homomorphism of commutative rings, taking 1 to 1, and sending any element $s \in S$ to a unit. Then there exist a unique homomorphism $f : S^{-1}R \longrightarrow R'$ such that $\varphi = f \circ \iota$.*

Exercise 3. *Let $R = \mathbf{C}[x, y]/(xy)$, and let $S = \{x^n\}_{n \geq 0}$. Describe the ring $S^{-1}R$.*

Exercise 4 (Exc. 16, p. 257)**.** *Let $x^4 - 16$ be an element of the polynomial ring $\mathbf{Z}[x]$, and let $E = \mathbf{Z}[x]/(x^4 - 16)$.*

(1) *Find a polynomial of degree ≤ 3 that is congruent to $7x^{13} - 11x^9 + 5x^5 - 2x^3 + 3$ modulo $(x^4 - 16)$.*
(2) *Prove that $x - 2$ and $x + 2$ are zero divisors in E.*

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