

Exercise IV

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Let R be commutativ and with 1.

Exercise 1

We let $R[x]$ denote the polynomial ring in the variabel x over R , and we let $R[[x]]$ denote the power series ring. Show that we have isomorphisms of R -modules

$$R[x] \cong \bigoplus_{i \geq 0} R \quad \text{and} \quad R[[x]] \cong \prod_{i \geq 0} R.$$

Exercise 2

Show that we have natural ring-homomorphisms

$$R[x] \otimes_R R[y] \longrightarrow R[x, y] \tag{1}$$

$$R[[x]] \otimes_R R[[y]] \longrightarrow R[[x, y]]. \tag{2}$$

Show furthermore that (1) is an isomorphism, whereas (2) is not surjective.