# LINEAR ALGEBRA 5B1307, JANUARY 2002

## Homework 2; 02/02/07

**1.** Prove that if $P \in \mathcal{L}(V)$; $P^2 = P$; and any of two conditions below is satisfied, then $P$ is an orthogonal projection.

(1) every vector in $nullP$ is orthogonal to every vector in $rangeP$;
(2) $\|Pv\| \leq \|v\|$ for all $v \in V$.

**2.** Suppose $T \in \mathcal{L}(V)$ and $U$ is a subspace of $V$. Prove

(1) $U$ is invariant under $T$ if and only if $P_U T P_U = T P_U$.
(2) $U$ and $U^\perp$ are both invarinat under $T$ if and only if $P_U$ and $T$ commute.

**3.** Consider the operator in $\mathbb{R}^3$ which has the matrix

$$\begin{pmatrix} 13 & 14 & 4 \\ 14 & 24 & 18 \\ 4 & 18 & 29 \end{pmatrix}$$

in the standard basis.

(1) Does it have a square root? Justify the answer.
(2) Find the square root if it exists.

**4.** Show that *dim range* $T$ is equal to the number of non-zero singular values of $T$.

**5.** Show that $S$ is an isometry if and only if all singular values of $S$ are equal to 1.

**6.** Consider $T_1, T_2 \in \mathcal{L}(V)$. Prove that $T_1$ and $T_2$ have the same singular values if and only if there exist isometries $S_1, S_2$ such that $T_1 = S_1 T_2 S_2$.

**7.** Consider the operator $T \in \mathcal{L}(\mathbb{R}^4)$ which has the matrix

$$\begin{pmatrix} 10 & -9 & 0 & 0 \\ 4 & -2 & 0 & 0 \\ 0 & 0 & -2 & -7 \\ 0 & 0 & 1 & 2 \end{pmatrix}$$

in the standard basis. Compute its Jordan form and the orthonormal basis in which $M(T)$ has this form.

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