

LINJÄR ALGEBRA 5B1307

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inga hjälpmedel är tillåtna

1. Let V be a finite dimensional vector space, $T \in \mathcal{L}(V)$. Give definitions of an *eigenvector*, *eigenspace*, *generalized eigenvector*, *space of generalized eigenvectors* of T .

In case when V is defined over \mathbb{C} , describe how to decompose V into a direct sum of non-trivial subspaces invariant under T .

2. Let $T \in \mathcal{L}(\mathbb{C}^5)$ be given by the formula

$$T(z_1, z_2, z_3, z_4, z_5) = (2z_1 + z_2 + z_3 + z_4, 2z_2, 2z_3 + z_4, z_4 + z_5, -z_2 - z_3 - z_4).$$

Find

- (1) the matrix of T in the standard basis;
- (2) the characteristic polynomial of T ;
- (3) the minimal polynomial of T ;
- (4) the Jordan form of T ;
- (5) a Jordan basis for T .

3. Find a polar decomposition of the operator $T \in \mathcal{L}(\mathbb{C}^3)$ if the following is known:

- (1) the matrix of T in the standard basis is $\begin{pmatrix} 2 & 4 & 2 \\ 3 & 2 & 0 \\ 0 & 2 & 5 \end{pmatrix}$;
- (2) the singular values of T are $\{1, 4, 7\}$.