Absolutely continuous invariant measures and superstable period orbits; weak*convergence of natural measures.

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Abstract. Consider the quadratic family $x \mapsto 1 - ax^2$. For most a-values between 0 and 2 there is a unique probability measure μ_a describing the asymptotic distribution under iteration of Lebesgue almost all initial points x. If $f_{\hat{a}}$ has a periodic attractor, then $\mu_{\hat{a}}$ consists of point-masses on the attractor and the mapping $a \mapsto \mu_a$ is weak* continuous at $\hat{a}$. We show that for a large class of maps, with an absolutely continuous measure μ describing the dynamics, there exists a sequence of nearby maps with stable periodic orbits, such that the measures supported on these periodic orbits converge weakly to the measure μ. In general, however, the measures do not depend continuously on the parameter. In proving our theorem, we review the proof of the Yakobson-Benedicks-Carleson Theorem.