# Aims versus Expectations – a Swedish study of problems related to the transition from secondary to tertiary education in mathematics

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*Abstract.* For a number of years now there has been reports that beginning science and engineering students at Swedish universities are having difficulties in passing their mathematics courses. Several studies involving diagnostic tests given to such students report a decline in student knowledge in areas generally considered to be important prerequisites of university mathematics. In order to describe, understand and explain the situation, we have performed a study involving not only the university perspective but also the perspective of students and secondary school teachers. We pinpoint areas of mathematics that constitute a *gap of content*, i.e. areas that are not covered in secondary school but nevertheless at university are treated as if they had already been covered. We also find a *clash of cultures*, i.e. a discrepancy in views of what constitutes important mathematical knowledge. Our method using questionnaires as well as course materials and official documents allows for a detailed description of how the goals of secondary school mathematics education are implemented in classroom practice, and how they compare with the expectations on newcomers in the classroom practice at the university.

## *Introduction*

Problems in the transition from secondary to tertiary level in mathematics have been a recurrent issue in Sweden during the last decades. For a number of years there have been reports that beginning science and engineering students at Swedish universities are having difficulties in passing their initial mathematics courses. At several of the main technical universities in Sweden, longitudinal studies in the form of pre-tests to newcomer students have been carried out (e. g. Bylund & Boo, 2003; Brandell, 2007). They all report on a considerable decline in student knowledge in areas generally considered to be important prerequisites of university mathematics. Also several governmental commissions have been assigned in order to investigate the problematic situation for math education in Sweden on different levels and to suggest measures to be taken (e.g. National Agency for Education, 1998; National Agency for Higher Education, 1999; Mathematics Delegation, 2004; National Agency for Higher Education, 2005; ).

In order to better describe, understand and explain the problems concerning the secondary-to-tertiary transition, we have performed a study involving not only the university perspective but also the perspectives of students and secondary school teachers. The goal was to compare the *aims* of mathematics education in Swedish upper secondary school with the *expectations* on the new students held by the tertiary level. The same approach was recently taken in a British study, and structural problems similar to the Swedish ones were identified (Hoyles, Newman & Noss, 2001).

A gap between secondary and tertiary level has been identified in several other studies from different countries. Results similar to the Swedish situation even on a detailed level can be found in recent studies from Canada (Kajander & Lovric, 2005) and the Netherlands (Heck & van Gastel, 2006). Transition problems in Hong Kong are described in (Hing, 2005). The situation in France and in Quebec some ten years ago is discussed in (Guzman, Hodgson, Robert & Villani, 1998). An international overview is given in (Selden, 2005).

Some aspects of recent reforms of the Swedish educational system relevant for this discussion is given in a parallel paper (Brandell, Hemmi & Thunberg, 2008) where also the investigations described in this paper is given a partially overlapping description. For an overview of the Swedish university system with two extensive structural and organisational reforms in 1977 and 1993 and the consequences for the mathematics departments, see Tengstrand (2001).

## Methodology

As mentioned above, the goal of the study here described was to compare the aims of upper secondary mathematics education with the expectations on newcomer student held by the tertiary level, represented by The Royal Institute of Technology (KTH) in Stockholm. In order to achieve this, the following investigations were performed.

*Tertiary level expectations* were identified from the literature and the exercise sets students were encouraged to use for preparation before entering KTH. More precisely we studied the web-based material *KTH Sommarmatematik* (KTH Summer mathematics) all newcomers were encouraged to study during summer before they start their studies (Johnsson, 2004) and the text book used during a non-compulsory preparatory course during the first two weeks at KTH (Dunkels et al, 2002). By definition, this material was regarded as most important for newcomers from the university's point of view. From the condensed presentation it also was obvious that this was material intended as a refresher of mathematics already studied in secondary school. This material thus offered a description of tertiary level expectations, in terms of central concepts, techniques and typical problems students were supposed to be familiar with (Thunberg & Filipsson, 2005b).

Mathematical knowledge consists of a number of competencies. We emphasize that our objects of study are only those explicit prerequisites found in these refresher courses. Thus we only came to consider skills and understanding of fundamental concepts within the following areas: numerical computations, algebraic simplifications, equation solving, and basic knowledge of some elementary functions. Obviously there are many other expectations held by the tertiary level which the students meet later on in their studies.

Furthermore we only consider the aims of the secondary level within those areas of mathematical knowledge that thus have been identified as explicit expectations from the tertiary level – our investigation is by no means an evaluation of the upper secondary curriculum and teaching practise as a whole.

*The newcomers' performance and pre-knowledge versus university expectations* was examined through various methods.

– A group of approximately 100 new engineering students at KTH were asked to record and comment on their experiences of the preparatory course given during the first two weeks, especially noting when they encountered concepts, techniques or problem types that they found difficult or unfamiliar. Answers were received from 36 students representing different backgrounds from upper secondary school. Those who did answer had slightly higher grades in mathematics from upper secondary school than the whole group, thus students' difficulties might in fact have been underestimated by this part of the project (Thunberg & Filipsson, 2005c).

- University teachers teaching the above-mentioned preparatory course were also asked to note how they experienced the new students' work with the course. Here, answers were obtained from 29 teachers out of group of 46 (Thunberg & Filipsson, 2005c).
- In a questionnaire, secondary school mathematics teachers were asked to grade how well prepared typical students would be after graduation from upper secondary school to handle various typical problems from KTH's preparatory course. The questionnaire was sent to 90 teachers in the Stockholm area, answers were obtained from 19 of them, representing 10 different schools (Thunberg & Filipsson, 2005d). We will describe the questionnaire and its outcome more in detail below.
- Students' solutions to written exams at the university were examined closely, looking for typical mistakes and misunderstandings. Two sets of student solutions were examined. The first one consisted of about 50 corrected exams from the preparatory course (Cronhjort 2005). The second set was taken from a first semester exam in one-variable calculus, where about 150 students' solutions to three selected problems were examined (Enström & Isaksson 2005).

*The aims of upper secondary school mathematics* were inferred from the compulsory national tests. Two rounds of national tests from 2002 and 2005, each consisting of four different tests for different levels in upper secondary school, have been made public[1]. These were our objects of study, and their mathematical content in areas that previously had been detected as problematic in the secondary-tertiary transition was compared with tertiary level expectations (Thunberg 2005).

## *Summary of results*

The different parts of our investigation mentioned above gave us a coherent picture of the difficulties for students in the transition from upper secondary school to university level in mathematics. We observed a *gap of content* as well as a *clash of cultures*. Examples will be given below, where we describe the questionnaire to the upper secondary school teachers and its outcome in more detail.

## Gap of content

There are several areas of mathematics that the students are expected to be well familiar with when they start their studies at the university but which are not part of the upper secondary school curriculum. We now describe the main constituents of this gap.

- *General knowledge about functions.* From the exercises used in the refresher courses at KTH one can see that *piecewise defined functions* and *composition of functions* are expected to be concepts familiar to the newcomers at the university, but from the answers in the questionnaire to the upper secondary teachers we know that neither of them can be considered as well known to the typical student leaving upper secondary

[1] National tests for *Mathematics A* are available http://www1.lhs.se/prim/matematik/tidigare_kurs_a.html , and tests for *Mathematics B – D* can be found at http://www.umu.se/edmeas/np/information/np-tidigare-prov.html .

school. One university teacher also points out that students' abilities to draw accurate sketches of elementary functions are below what is expected.

– *Analytic geometry*. Although the Theorem of Pythagoras of course is well known to students in upper secondary school, few of them are familiar with the distance formula or the equation of a circle in its general form – this is not part of the standard curriculum. When entering the university however the students are expected to be well familiar with the equation of a circle, and also to be prepared to generalise to general conics in the plane.
– *The absolute value function* is according to the upper secondary teachers not part of the upper secondary curriculum, still this is treated in the refresher course as something that just needs a quick review. Many of the newcomer students mention equations and inequalities involving absolute values as a major difficulty, this is also noted by the university teachers.
– *Inequalities.* In upper secondary school students solve linear inequalities in two variables analytically. Nonlinear inequalities are solved graphically using the calculator. Still, the university expects students to be able to solve polynomial inequalities using factorization and sign-tables.

Our investigations also indicate misconceptions at the university level about how and with which learning outcomes certain topics are treated in upper secondary school. This includes *algebraic techniques* like "completing the square", *techniques for solving special types of equations* and *logarithms.* We will discuss this further in the next section.

## Clash of cultures

There are also different views on what it means to learn mathematics at secondary and tertiary levels. What aspects are in focus? What are the desired outcomes of a learning process? These differences are most clearly visible in the analysis of mistakes and misunderstandings in students' solutions to examination tasks and in the comparison of the requirements in national tests for upper secondary school and the expectations from tertiary level. The comparison between the requirements on national tests and in the university preparatory course is described in (Brandell, Hemmi & Thunberg, 2008).

Let us just say a word of caution before we go on: It is really not possible to identify a single common upper secondary attitude concerning these matters. There seems to be different subcultures with different views living in parallel in the upper secondary world. Below, we will compare university level expectations with requirements on national tests for upper secondary level. These national tests put great emphasis on testing students understanding of concepts and of their problem solving skills, while computing skills and knowledge of formulas and mathematical theory is hardly required at all. On the other hand, recent research indicates that teacher made tests typically have much more emphasis on computing skills (Boesen, 2006) and that most exercises in the more common textbooks are tasks that may be solved by a standard method presented on the page before. (Lithner, 2000). Still we find that typical weaknesses in university students' pre-knowledge do agree with areas not emphasised in the national tests.

– *Routine skills in arithmetic and algebraic computations* are considered as an absolutely necessary ingredient when learning mathematics at university level, and newcomer students are suddenly expected to handle much more complicated expressions and computations than they have met before. According to teachers at the university, students get confused when confronted with tasks where all necessary steps are not clear from the beginning. Computing errors of various kinds, often indicating a lack of routine, were also frequently observed in the examined student solutions to university exams.

- *Knowledge about the logarithm.* University expectations include a working knowledge of the logarithmic function and the laws of the logarithm. In upper secondary school on the other hand, logarithms are mainly used in connection with growth problems as a tool to solve for the exponent using the calculator, whereas the laws of the logarithm and the properties of the logarithmic function are not stressed at all.
- *Tables and calculators vs knowledge of formulas and standard identities.* In first year mathematics at the university, students are suddenly expected to work without calculators and tables, in upper secondary school these tools are almost always at hand. Routine skills and knowledge of formulas and theorems (procedural knowledge) are at the university level considered necessary for the understanding of concepts and theory, as well as important tools in problem solving. This seems to be in sharp contrast with the upper secondary level goals, at least as they manifest themselves in national tests, where calculations and formulas rather seem to be regarded as difficulties that hinders students from a deeper understanding and thus should be avoided.

Let us comment on the last item. When analysing common errors and mistakes among the new university students, one finds that a common mistake is the use of incorrect formulas. We propose the following explanation to this fact: In upper secondary school, students could always rely on tables with all useful formulas listed. There was never any need to internalise formulas as part of a larger whole or to develop strategies to conjecture, test and falsify or prove formulas. A particular example of this would be *trigonometric functions* which even though extensively studied in upper secondary school, still are considered as a major difficulty when entering the university.

## *The questionnaire to secondary school teachers*

We devote this section to presenting the results of the above-mentioned questionnaire given to upper secondary school teachers, in which they were asked to categorize exercises given to beginning university students at KTH. These exercises were originally designed in order to help students refresh knowledge and skills they (supposedly) had previously acquired in secondary school. The purpose of the questionnaire was to determine to what extent these refresher exercises dealt with material that the students had worked on before and whether the demands of KTH were compatible with the goals of upper secondary school. The exercises were taken from two sources, the web-based material *KTH Sommarmatematik* (Johnsson, 2004), and the Swedish text book *Mot bättre vetande i matematik* (Dunkels et al, 2002) used in the preparatory course at KTH, taken by beginning engineering students during their first two weeks.

### The questionnaire

In the questionnaire we asked the upper secondary school teachers to classify a number of exercises with respect to how well prepared two different categories of students be would be after graduating from upper secondary school to handle these exercises. The student categories we had in mind were

| | |
|---|---|
| Category G: | students with a low but passing grade |
| Category VG+: | students with a fairly high, if not top, grade |

This reflects the present Swedish grading system in which there are three passing grades: G (pass), VG (pass with distinction) and MVG (pass with special distinction). For entering engineering studies at KTH grade G from the appropriate courses suffices. Approximately one fifth of the newcomer engineering students at KTH has grade G from the last required course in upper secondary.

The teachers were asked to categorize to what extent a typical student from each category would be able to solve a given exercise. There were four categories, I-IV, as follows:

Category I: The student has learnt to master this kind of exercise in secondary school very well and is able to solve this kind of exercise without help and without any refresher course.

Category II: The student has learnt to solve this kind of exercise in secondary school, but will probably need a short refresher course before he or she is able to solve this kind of exercise without help

Category III: The student has to some extent worked on this kind of exercise in secondary school and is probably able to follow a given solution, but has not reached the level where he or she is able to solve this kind of exercise without help.

Category IV: The student has never worked on this kind of exercise in secondary school.

*Example:*

Compute $\frac{1/3+1/4}{1/5}$.

| | **I** | **II** | **III** | **IV** |
|---|---|---|---|---|
| **G** | | **X** | | |
| **VG+** | **X** | | | |

A teacher checking the boxes as in the example holds the opinion that this is a category II exercise for the G student (meaning that he or she is able is to solve it without help after a short refresher course) and a category I exercise for the VG+ student (meaning that he or she is able to solve it without help and without having to go through any refresher course first).

The questionnaire was sent out to 90 upper secondary school teachers. 19 teachers from ten different schools answered.

## Conclusions and results

As explained above, the questionnaire to secondary school teachers is one of five different investigations completed within the framework of the project “Aims vs. Expectations”. The following general conclusions that can be drawn from the questionnaire.

1. A great part of the mathematics that the university expects that the students have learned to master already in secondary school either has not been taught there at all or has been studied there with goals and methods that are not compatible with the expectations of the university level, and hence needs to be studied again at university.
2. The computational complexity of the KTH refresher exercises is way above the level of complexity that the students have met at secondary school.
3. The difference between the two student categories, G and VG+, is generally by the teachers considered to be very large.
4. When commenting on their answers, the teachers often write that this kind of exercises at secondary school never would be solved without tables or calculators.

Similar conclusions can be drawn from all of the other investigations in the project. In an appendix we present the complete questionnaire along with compilations of the comments from the teachers, but let us now indicate the most important results.

It is interesting to note which exercises are considered by the teachers to be the most difficult ones. These are:

Exercise 33. Determine the geometric significance of the equation $x^2 + 2x + y^2 - 4y + 4 = 0$ and sketch the corresponding curve.

Exercise 16. Solve the equation $|x-3| + x = 5$.

Exercise 32. Sketch the curve $(x-2)^2 + y^2 = 1$.

Exercise 19. Determine, without approximations, which is the biggest number, $\sqrt[3]{7}$ or $\sqrt[4]{13}$.

Exercise 22. Solve the equation $5^{3x}2^x = \sqrt{250}$.

Exercise 28. Determine the domain of definition of the function $f(x) = \sqrt{x^2 - 3x + \frac{7-x^3}{x-1}}$.

The fact that precisely these exercises are considered to be the most difficult ones is by no means a coincidence, as can be seen in the comments given by the teachers:

- The equation of the circle is not studied in secondary school.
- Absolute values are not studied in secondary school
- No training in the laws of the logarithm is given at secondary school.
- This level of computational complexity is never encountered in secondary school.

Here is a closer analysis of the answers to the questionnaire.

**Numerical computations** (exercise 1-4). Not even one of these exercises is considered to be so simple that the G student is able to solve it without any extra teaching or training. At least a short review is needed for the G student to be able to do fractions and positive integer exponents. When it comes to negative integer exponents, and fractions where there is a root sign in the denominator, a short refresher is not enough: these exercises are placed in category III which means that the G student needs to study this area carefully before being able to solve such exercises. The VG+ student is considered to have learnt this area fairly well in secondary school.

**Algebraic simplifications** (exercise 5-9). Only the first exercise is considered to be within reach (category II) for the G student. All other exercises are placed in category III or IV. The VG+ student can solve these exercises after a short refresher course.

**Equations and inequalities** (exercise 10-17). Only the quadratic equation is considered to be of a kind that is thoroughly practised in secondary school. The rest are more difficult. Solving simple rational inequalities seems to be a subject unknown to the G student, as are absolute values and root equations. All of these are placed in category IV. For the VG+ student, absolute values are placed in category IV and the other exercises in category III.

**Exponents and logarithms** (exercise 18-22). Not even the simplest of these five exercises (compute $\sqrt[4]{16/81}$ ) is considered to be simple enough for the G student to solve after a little practice. Students do not get enough practice in the laws of the logarithm in secondary school. This material needs to be studied thoroughly at university.

**Trigonometry and functions** (exercise 23-33). Only two exercises are placed in category II for the G student, the rest are in category III or IV. In spite of the fact that a lot of time is spent on trigonometry in upper secondary school, the G student cannot be expected to solve simple trigonometric equations even after a short refresher course. For the VG+ student the situation is a little better, but at least part of this material needs a lot of attention at university.

For a typical G student, this is how the material is categorized by the teachers:

**Category I.** Nothing.

**Category II.** Fractions. Positive integer exponents. Quadratic equations.

**Category III.** Completing the square. Rational equations. Trigonometric equations. Composition of functions. Negative integer exponents.

**Category IV.** Inequalities. Absolute values. Root equations. Equations involving logarithms. The equation of the circle. Fractions with a sqare roots in the denominator.

For a G student none of the exercises are placed in category I and only 7 in category II. 12 are placed in category III and 14 in category IV. This means that 80 percent of the exercises are considered to be so difficult that they are not suitable for a short refresher course – they demand a more thorough study at university.

For a typical VG+ student, this is how the material is categorized by the teachers:

**Category I.** Fractions. Positive integer exponents. Quadratic equations.

**Category II.** Completing the square. Rational equations. Trigonometric equations. Composition of functions. Negative integer exponents. Double fractions or fractions with a root sign in the denominator.

**Category III.** Inequalities. Root equations. Equations involving logarithms.

**Category IV.** Absolute values. The equation of the circle.

For a VG+ student 8 of the exercises are placed in category I, 14 are placed in category II, 8 in category III and 3 in category IV. Even for this student about one third of the exercises are too difficult to be suitable for a refresher course.

## *Discussion*

The discrepancy between the aims of upper secondary school and the expectations of the university could be described as a matching problem between two sequential educational systems. As described above, there is a *gap of content* as well as a *clash of cultures* that the students have to deal with when passing from the secondary to the tertiary level.
Looking at changes in the Swedish educational system during the last decades one finds several possible contributing causes.

- Gradually during the last decades, Swedish primary and secondary education has through political reforms developed into one of the least differentiated school systems in the world. One of the guiding goals has been that all students leaving upper secondary school, from vocational programs as well as theoretical programs, should fulfil the basic admittance demands for university studies. As a consequence, all students take the same mathematics courses up until and including the first course on the upper secondary level, i.e., until the age of 16. This leaves very little room for more specialized and advanced courses for students studying natural science and technology in order to prepare for engineering or science programs at the university.
- There has been a gradual drift where certain pieces of mathematics traditionally taught in upper secondary school have disappeared from the implemented curriculum. This is the case with conic sections in general and the equation of the circle in particular, the absolute value function and the laws of the logartithm.
- The tertiary level has been ignorant of the above mentioned changes in upper secondary school.
- Through a mixture of secondary effects of other changes and deliberate actions at the university level, the entrance requirements in mathematics at KTH and most other technical universities in Sweden have been lowered substantially, most often without adequate changes in the first year curriculum.

These problems have indeed gained quite a lot of attention in Sweden during the last years, and several actions on different levels have been taken that might reduce these particular problems over the next, say, ten years.

- The current government has initiated a reform of upper secondary school that will lead to more specialized courses in mathematics on the science program in upper secondary school.
- Universities are more aware of the situation, and actions like restoration of entrance qualifications and reformation of the first year courses have been taken.
- Short term remedies, like temporary bridging courses offered as voluntary preparation for the university, have been introduced.
- Common pedagogical meetings and projects with teachers from both levels are taking place, partly as a consequence of governmental initiative.

Finally we want to point out that two of the investigations carried out within the framework of our project are based on questionnaires, given to secondary school teachers and university students and teachers. Those answering the questionnaires have interpreted the questions therein, and we have, in turn, interpreted and categorized their answers. A substantial number of recipients chose not to answer the questionnaire sent to them. Therefore, one needs to be careful in drawing statistical conclusions based on this material. However, the questionnaires constitute only a part of our study, and we feel confident in drawing the conclusions previously mentioned, since the results of the questionnaires are completely consistent with all of the other investigations within the project.

## *Appendix. The complete questionnaire*

Here we present the complete questionnaire along with the answers from the teachers. The numbers in the tables below indicate how many teachers have placed this particular exercise in the respective categories. Under each table we summarize the comments given by the teachers answering the questionnaire. When chosing what exercises to put in the questionnaire we deliberately avoided the most difficult ones as well as the easiest ones. We simply tried to include typical exercises from each area considered.

1. Compute $\frac{1/3+1/4}{1/5}$.

| | **I** | **II** | **III** | **IV** |
|---|---|---|---|---|
| **G** | **1** | **9** | **8** | **0** |
| **VG+** | **15** | **2** | **1** | **0** |

Comments: Many of the teachers say in their comments to this exercise that elementary computations with fractions are difficult for many students.

2. Compute $\frac{(3^{-4})^{(-5)}}{243^3}$.

| | **I** | **II** | **III** | **IV** |
|---|---|---|---|---|
| **G** | **0** | **6** | **11** | **1** |
| **VG+** | **7** | **10** | **1** | **0** |

Comments: The teachers comment that this exercise is quite hard, in spite of the fact that the students have had some practise with the laws of the exponent. Negative exponents is a problem, as is factoring the number 343. Maybe the VG+ student can do this, but not the G student. For many students a calculator is needed to solve this exercise.

3. Write in a form where there is no square root in the denominator: $\frac{1}{1-(\sqrt{3}-1)^2}$.

| | **I** | **II** | **III** | **IV** |
|---|---|---|---|---|
| **G** | **0** | **1** | **7** | **10** |
| **VG+** | **3** | **8** | **7** | **0** |

Comments: According to the teachers, the students are not familiar with this kind of exercise, a VG+ student might be able to solve it anyway.

4. $\sqrt{2^{16}}$ equals: $2^4$ or $2^6$ or $2^8$?

| | I | II | III | IV |
|---|---|---|---|---|
| **G** | **4** | **10** | **5** | **0** |
| **VG+** | **17** | **2** | **0** | **0** |

Comments: Teachers say, it is not clear to the students that taking the square root of a number is equivalent to raising it to the power ½.

5. Simplify the expression $(p+q)(p-q)-(p-q)^2$.

| | I | II | III | IV |
|---|---|---|---|---|
| **G** | **3** | **13** | **3** | **0** |
| **VG+** | **18** | **1** | **0** | **0** |

Comments: Teachers remark that the G student might have problems in getting the correct sign.

6. Put the following expression on a common denominator: $\dfrac{a+2b}{1+\dfrac{1}{cx-1}}+\dfrac{2a+b}{1-\dfrac{1}{cx-1}}$

| | I | II | III | IV |
|---|---|---|---|---|
| **G** | **0** | **0** | **8** | **10** |
| **VG+** | **1** | **10** | **6** | **1** |

Comments: According to the teachers, this kind of exercise is rarely met at secondary school and the complexity of the expression will scare at least the G student.

7. Simplify the expression: $\dfrac{a^3+ab^2}{a^3-ab^2}\cdot\dfrac{a^2-ab}{a^2+ab}$

| | I | II | III | IV |
|---|---|---|---|---|
| **G** | **0** | **2** | **14** | **3** |
| **VG+** | **7** | **10** | **2** | **0** |

Comments: The teachers suspects that many students will do the multiplication first and then will have difficulties factoring out.

8. Express R in terms of the other variables: $\dfrac{1}{R}=\dfrac{1}{R_1}+\dfrac{1}{R_2}+\dfrac{1}{R_3}$.

| | I | II | III | IV |
|---|---|---|---|---|
| **G** | **0** | **3** | **10** | **6** |
| **VG+** | **8** | **6** | **5** | **0** |

Comments: Teachers say, the VG+ student will be able to solve this one if he or she is not required to simplify the answer.

9. Complete the square: $3x^2 - 24x + 40$.

| | I | II | III | IV |
|---|---|---|---|---|
| **G** | **0** | **0** | **11** | **8** |
| **VG+** | **0** | **12** | **5** | **2** |

Comments: Teachers point out that this is hardly part of the curriculum at secondary school and the suspect that many students will divide by 3 and solve a quadratic equation.

10. Solve the equation $4x^2 - 4x + 1 = 0$.

| | I | II | III | IV |
|---|---|---|---|---|
| **G** | **6** | **11** | **2** | **0** |
| **VG+** | **19** | **0** | **0** | **0** |

Comments: This ought to be ok for most students, the teachers say, although some of them will be bewildered by the double root.

11. Solve the equation $15(x-2)^2 = 16(x-2)$.

| | I | II | III | IV |
|---|---|---|---|---|
| **G** | **2** | **10** | **7** | **0** |
| **VG+** | **12** | **7** | **0** | **0** |

Comments: Teachers foresee that some students will destroy the factorisation given and use the quadratic formula and some will lose a solution when dividing out by $x - 2$.

12. Solve the inequality $x^3 - 3x + 2x > 0$.

| | I | II | III | IV |
|---|---|---|---|---|
| **G** | **0** | **1** | **5** | **13** |
| **VG+** | **2** | **5** | **10** | **2** |

Comments: Teachers remark that this is not treated in upper secondary school.

13. Solve the inequality $\frac{1}{x} < 2x - 1$. Make sure you have a 0 on one side of the inequality sign first.

| | I | II | III | IV |
|---|---|---|---|---|
| **G** | **0** | **0** | **6** | **13** |
| **VG+** | **1** | **8** | **8** | **2** |

Comments: Teachers remark that this is not treated in upper secondary school. Some students might be able to do graphic or numeric solutions, perhaps using their calculator.

14. Solve the equation: $\frac{3}{2x}+\frac{5}{6x}=\frac{2}{9}+\frac{5}{3x}$.

| | I | II | III | IV |
|---|---|---|---|---|
| **G** | **0** | **4** | **12** | **3** |
| **VG+** | **9** | **8** | **2** | **0** |

Comments: This is hard, some teachers comment.

15. Solve the equation: $\frac{x}{x+2a}+\frac{2}{x-3}=1$.

| | I | II | III | IV |
|---|---|---|---|---|
| **G** | **0** | **1** | **7** | **11** |
| **VG+** | **3** | **8** | **7** | **1** |

Comments: Teachers expect the parameter to be confusing to the students.

16. Solve the equation: $|x+3|+x=5$.

| | I | II | III | IV |
|---|---|---|---|---|
| **G** | **0** | **0** | **0** | **19** |
| **VG+** | **0** | **0** | **6** | **13** |

Comments: Teachers remark that this is not treated in upper secondary school.

17. Solve the equation $\sqrt{x}-1=\sqrt{x-9}$.

| | I | II | III | IV |
|---|---|---|---|---|
| **G** | **0** | **0** | **5** | **14** |
| **VG+** | **1** | **8** | **9** | **1** |

Comments: Some students may be able to do a graphic solution, otherwise this is not treated in secondary school, according to the teachers.

18. Compute $\sqrt[4]{\frac{16}{81}}$.

| | I | II | III | IV |
|---|---|---|---|---|
| **G** | **1** | **6** | **9** | **3** |
| **VG+** | **12** | **5** | **2** | **0** |

Comments: Teachers remark that students are not used to working with third and fourth roots.

19. Determine, without approximations, which is the biggest number: $\sqrt[3]{7}$ or $\sqrt[4]{13}$ .

| | I | II | III | IV |
|---|---|---|---|---|
| **G** | **0** | **0** | **1** | **18** |
| **VG+** | **0** | **3** | **10** | **6** |

Comments: Unusual kind of exercise in secondary school, according to the teachers. Would be solved using a calculator.

20. Compute $\ln\frac{1}{e} + 2\ln\sqrt{e}$ .

| | I | II | III | IV |
|---|---|---|---|---|
| **G** | **0** | **2** | **9** | **8** |
| **VG+** | **4** | **8** | **6** | **1** |

Comments: Teachers remark that the students are not well trained in using the laws of the exponent and the laws of the logarithm. The combination of the two makes it even more difficult. Even $1/e = e^{-1}$ is tricky for most students.

21. Solve the equation: $\ln x + \ln(x+4) = \ln(2x+3)$ .

| | I | II | III | IV |
|---|---|---|---|---|
| **G** | **0** | **1** | **8** | **10** |
| **VG+** | **2** | **7** | **10** | **0** |

Comments: A non-existent kind of exercise in secondary school, according to the teachers. Logarithmic equations are not solved there at all.

22. Solve the equation $5^{3x}2^{x} = \sqrt{250}$ .

| | I | II | III | IV |
|---|---|---|---|---|
| **G** | **0** | **0** | **5** | **14** |
| **VG+** | **0** | **4** | **10** | **5** |

Comments: As above.

23. Determine the exact value of $\cos(13\pi + 35\pi/6)$ .

| | I | II | III | IV |
|---|---|---|---|---|
| **G** | **0** | **0** | **9** | **10** |
| **VG+** | **3** | **5** | **8** | **3** |

Comments: Teachers expect student difficulties in using the unit circle, radians, addtion formulas. Exact values of the sine and cosine function are not memorized. Tables and formula sheets would be needed.

24. Sketch in the same coordinate system the two curves: $y = \cos x$ and $y = \cos\frac{x}{2}$.

| | I | II | III | IV |
|---|---|---|---|---|
| **G** | **1** | **11** | **5** | **2** |
| **VG+** | **12** | **6** | **1** | **0** |

Comments: Teachers remark that the students are not used to sketching curves without using a graphing calculator.

25. Solve the equation $\cos 4x = 1/2$.

| | I | II | III | IV |
|---|---|---|---|---|
| **G** | **2** | **5** | **11** | **1** |
| **VG+** | **9** | **7** | **3** | **0** |

Comments: Teachers expect the G student will omit solutions (forgets +/- or periodicity) and the factor 4 in the argument makes this exercise extra difficult. Much time is devoted to trigonometric equations in secondary school, but nevertheless many students fail to acquire the skills needed to solve this kind of exercises. Tables and formula sheets would be needed.

26. Solve the equation: $\sin 2x - 4\cos x = 0$.

| | I | II | III | IV |
|---|---|---|---|---|
| **G** | **0** | **4** | **10** | **5** |
| **VG+** | **5** | **11** | **3** | **0** |

Comments: Teachers say, tables or calculators are used for this kind of exercise.

27. Show that the formula $\cos 3v = 4\cos^3 v - 3\cos v$ holds true for all $v$.

| | I | II | III | IV |
|---|---|---|---|---|
| **G** | **0** | **0** | **11** | **8** |
| **VG+** | **2** | **9** | **6** | **2** |

Comments: ”Show that…” is always a difficult exercise, the teachers remark. This kind of formula is taken from a table when needed.

28. Determine the domain of definition of the function $f(x) = \sqrt{x^2 - 3x + \frac{7 - x^3}{x - 1}}$.

| | I | II | III | IV |
|---|---|---|---|---|
| **G** | **0** | **0** | **2** | **16** |
| **VG+** | **0** | **3** | **10** | **5** |

Comments: Teachers point out that this level of computational complexity is never encountered in secondary school.

29. If $g(x) = \frac{(x+1)(x-1)}{x}$, compute $g(0.5)$.

| | I | II | III | IV |
|---|---|---|---|---|
| **G** | **7** | **8** | **3** | **0** |
| **VG+** | **16** | **2** | **0** | **0** |

Comments: This exercise should work out OK for most students, maybe the G student will end up with the wrong sign, the teachers say.

30. If $f(x) = x - \frac{1}{x}$ and $g(x) = 3x^2 + 1$, compute $g(x-1)$ and $f(g(x))$.

| | I | II | III | IV |
|---|---|---|---|---|
| **G** | **0** | **4** | **8** | **6** |
| **VG+** | **6** | **8** | **4** | **0** |

Comments: Teachers remark that compositions of functions are out of reach for most students.

31. Sketch the curve $y = \begin{cases} x+1, x<0 \\ 2x+1, x \geq 0 \end{cases}$.

| | I | II | III | IV |
|---|---|---|---|---|
| **G** | **2** | **2** | **5** | **9** |
| **VG+** | **6** | **4** | **6** | **2** |

Comments: Teachers point out that piecewise defined functions are not dealt with in secondary school.

32. Sketch the curve $(x-2)^2 + y^2 = 1$.

| | I | II | III | IV |
|---|---|---|---|---|
| **G** | **0** | **0** | **2** | **16** |
| **VG+** | **0** | **3** | **4** | **11** |

Comments: Teachers point out that the equation of the circle is not treated in secondary school.

33. Determine the geometric significance of the equation and sketch the corresponding curve: $x^2 + 2x + y^2 - 4y + 4 = 0$ .

| | **I** | **II** | **III** | **IV** |
|---|---|---|---|---|
| **G** | **0** | **0** | **0** | **16** |
| **VG+** | **0** | **0** | **2** | **14** |

Comments: Teachers point out that this kind of exercise is not encountered in upper secondary school .

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