1.

$$y'' + y = \frac{1}{\sin x}$$

$$y_h = C_1 \cos x + C_2 \sin x$$

$$y_p = u(x)\cos x + v(x)\sin x$$

$$\begin{pmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{pmatrix}\begin{pmatrix} u' \\ v' \end{pmatrix} = \begin{pmatrix} 0 \\ \dfrac{1}{\sin x} \end{pmatrix}$$

Systemdeterminanten = Wronskideterminanten = 1 .

$$\begin{cases} u' = \dfrac{\begin{vmatrix} 0 & \sin x \\ \dfrac{1}{\sin x} & \cos x \end{vmatrix}}{1} = -1 \\ v' = \dfrac{\begin{vmatrix} \cos x & 0 \\ -\sin x & \dfrac{1}{\sin x} \end{vmatrix}}{1} = \dfrac{\cos x}{\sin x} \end{cases}$$

$$\begin{cases} u = -x \\ v = \ln\left|\sin x\right| \end{cases}$$

$$y = C_1 \cos x + C_2 \sin x$$
$$-x\cos x + \sin x \ln\left|\sin x\right|$$

2.

$$f(x) = \begin{cases} -1 \;, & -\pi < x < 0 \\ 1 \;, & 0 \le x < \pi \end{cases}$$

f är en udda funktion.

Fourierserien är på formen: $\sum_{n=1}^{\infty} b_n \sin nx$.

$$b_n = \frac{2}{\pi}\int_0^{\pi} f(x)\sin nx\,dx = \frac{2}{\pi}\int_0^{\pi} 1\sin nx\,dx =$$

$$= \frac{2}{\pi}\left[\frac{-\cos nx}{n}\right]_0^{\pi} = \frac{2}{\pi n}(1-\cos n\pi) =$$

$$= \frac{2(1-(-1)^n)}{\pi n} = \begin{cases} 0 \;, & n = 2m \\ \dfrac{4}{\pi(2m+1)} \;, & n = 2m+1 \end{cases}$$

$$f \sim \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1-(-1)^n}{n}\sin nx = \frac{4}{\pi}\sum_{m=0}^{\infty}\frac{1}{2m+1}\sin(2m+1)x$$

3.

$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=u$$

$$\text{Ansätt}: u(x,y)=X(x)Y(y).$$

$$X'(x)Y(y)+X(x)Y'(y)=X(x)Y(y)$$

$$\frac{X'(x)}{X(x)}+\frac{Y'(y)}{Y(y)}=1$$

$$\frac{X'(x)}{X(x)}=1-\frac{Y'(y)}{Y(y)}=\text{konstant}=\lambda$$

$$\begin{cases} X'(x)-\lambda X(x)=0 \\ Y'(y)-(1-\lambda)Y(y)=0 \end{cases}$$

$$\begin{cases} X(x)=Ae^{\lambda x} \\ Y(y)=Be^{(1-\lambda)y} \end{cases}$$

$$u(x,y)=Ae^{\lambda x}Be^{(1-\lambda)y}=Ce^{\lambda x+(1-\lambda)y}$$