

$$1. \quad L^{-1} \frac{8}{s^2 + 6s + 25} = L^{-1} \frac{2 \cdot 4}{(s + 3)^2 + 4^2} = 2e^{-3t} \sin 4t$$

$$2. \quad \int_0^t f(t) + 4 \int_0^t f(\tau) d\tau = 1$$

$$F(s) + 4F(s) \frac{1}{s} = \frac{1}{s}$$

$$F(s) = \frac{1}{s + 4}$$

$$f(t) = e^{-4t}$$

3. $y'' + 4y = \delta\left(t - \frac{\pi}{2}\right), \quad y(0) = y'(0) = 0$

$$s^2 Y(s) - sy(0) - y'(0) + 4y = e^{-s\frac{\pi}{2}}$$

$$Y(s) = \frac{1}{s^2 + 4} e^{-s\frac{\pi}{2}} = \frac{1}{2} \frac{2}{s^2 + 2^2} e^{-s\frac{\pi}{2}}$$

$$y(t) = \frac{1}{2} U\left(t - \frac{\pi}{2}\right) \sin 2\left(t - \frac{\pi}{2}\right)$$