10.1.24.

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} y + x(x^2+y^2) \\ -x + y(x^2+y^2) \end{pmatrix}, \quad \mathbf{X}(0) = \begin{pmatrix} 4 \\ 0 \end{pmatrix}$$

Inför polära koordinater : $\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases}$.

$$\begin{cases} \dfrac{dr}{dt} = \dfrac{x}{r}\dfrac{dx}{dt} + \dfrac{y}{r}\dfrac{dy}{dt} \\ \dfrac{d\theta}{dt} = \dfrac{-y}{r^2}\dfrac{dx}{dt} + \dfrac{x}{r^2}\dfrac{dy}{dt} \end{cases}$$

$$\begin{cases} r^2 = x^2 + y^2 \\ \tan\theta = \dfrac{y}{x} \end{cases}$$

$$\begin{cases} 2r\,dr = 2x\,dx + 2y\,dy \\ (1+\tan^2\theta)\,d\theta = \dfrac{-y}{x^2}dx + \dfrac{1}{x}dy \end{cases}$$

$$\begin{cases} dr = \dfrac{x}{r}dx + \dfrac{y}{r}dy \\ d\theta = \dfrac{-y}{r^2}dx + \dfrac{x}{r^2}dy \end{cases}$$

$$\begin{cases} \dfrac{dr}{dt} = \dfrac{x}{r}(y + xr^2) + \dfrac{y}{r}(-x + yr^2) \\ \dfrac{d\theta}{dt} = \dfrac{-y}{r^2}(y + xr^2) + \dfrac{x}{r^2}(-x + yr^2) \end{cases}$$

$$\begin{cases} \dfrac{dr}{dt} = r^3 \\ \dfrac{d\theta}{dt} = -1 \end{cases}$$

$$\begin{cases} \displaystyle\int \frac{dr}{r^3} = \int dt \\ \displaystyle\int d\theta = \int -dt \end{cases} \quad \begin{cases} \displaystyle\frac{-1}{2r^2} = t + C_1 \\ \theta = -t + C_2 \end{cases}$$

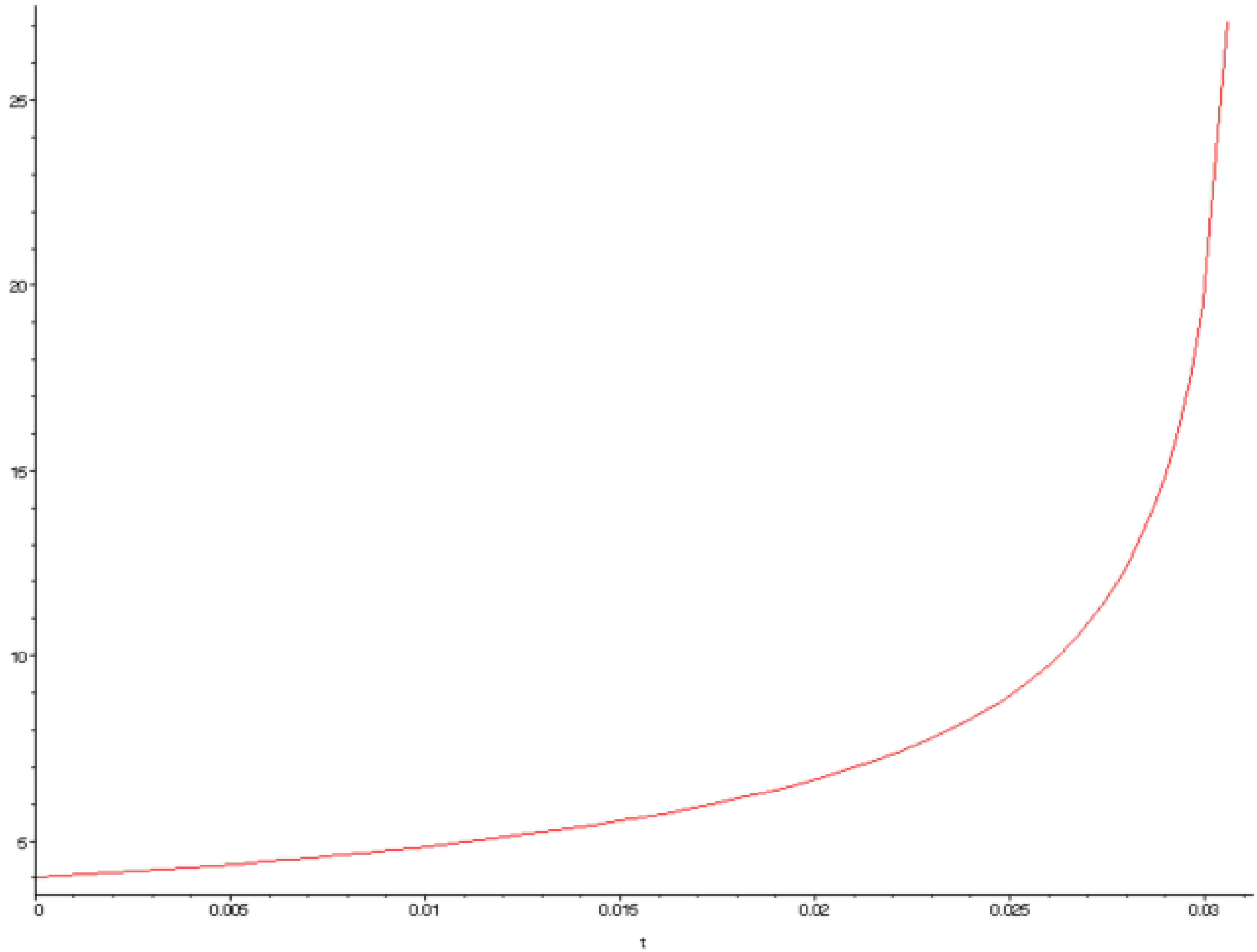
Villkoret $\mathbf{X}(0) = \begin{pmatrix} x(0) \\ y(0) \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}$ ger $\begin{pmatrix} r(0) \\ \theta(0) \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}$.

$$\begin{cases} \displaystyle\frac{-1}{32} = C_1 \\ 0 = C_2 \end{cases}$$

$$\begin{cases} \dfrac{-1}{2r^2} = t - \dfrac{1}{32} = \dfrac{32t-1}{32} \\ \theta = -t \end{cases}$$

$$\begin{cases} r = \dfrac{4}{\sqrt{1-32t}} \\ \theta = -t \end{cases}$$

Kurvan existerar för $t: 0 \to \left(\dfrac{1}{32}\right)^{-}$.



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