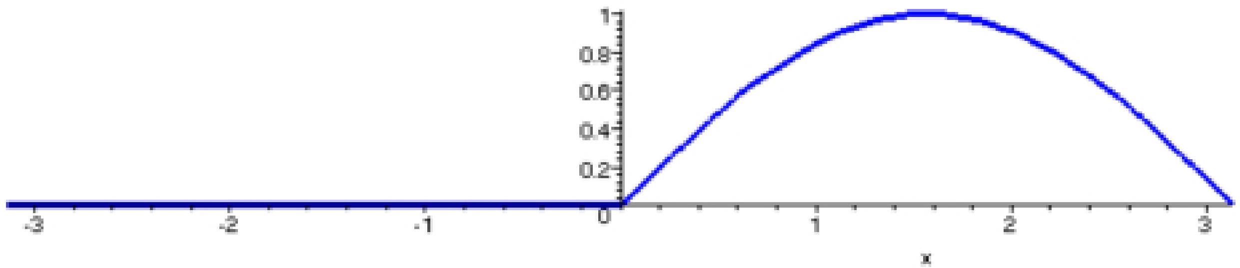
11.2.9.

$$f(x) = \begin{cases} 0 \quad , & -\pi < x < 0 \\ \sin x \ , & 0 < x < \pi \end{cases}$$



$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx = \frac{1}{\pi}\left\{ 0 + \int_0^{\pi} \sin x \cos nx\,dx \right\} =$$

$$= \frac{1}{\pi}\int_0^{\pi} \frac{1}{2}\left(\sin(n+1)x - \sin(n-1)x\right)dx =$$

$$= [n \neq 1] = \frac{1}{2\pi}\left[\frac{-\cos(n+1)x}{n+1} + \frac{\cos(n-1)x}{n-1} \right]_0^{\pi} =$$

$$= \frac{1}{2\pi}\left\{ \frac{1-\cos(n+1)\pi}{n+1} - \frac{1-\cos(n-1)\pi}{n-1} \right\} =$$

$$= \frac{1}{2\pi}\left\{\frac{1-(-1)^{n+1}}{n+1} - \frac{1-(-1)^{n+1}}{n-1}\right\} =$$

$$= \frac{1-(-1)^{n+1}}{2\pi}\left\{\frac{1}{n+1} - \frac{1}{n-1}\right\} = -\frac{1+(-1)^{n}}{\pi(n^{2}-1)}$$

$$a_0 = \frac{2}{\pi}$$

$$a_1 = \frac{1}{\pi}\int_0^{\pi}\frac{1}{2}\sin 2x\,dx = 0$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx = \frac{1}{\pi}\left\{0 + \int_0^{\pi}\sin x \sin nx\,dx\right\} =$$

$$= \frac{1}{\pi}\int_0^{\pi}\frac{1}{2}(-\cos(n+1)x + \cos(n-1)x)dx =$$

$$= [n \neq 1] = \frac{1}{2\pi}\left[\frac{-\sin(n+1)x}{n+1} + \frac{\sin(n-1)x}{n-1}\right]_0^{\pi} = 0$$

$$b_1 = \frac{1}{\pi}\int_0^{\pi}\sin^2 x dx = \frac{1}{\pi}\int_0^{\pi}\frac{1-\cos 2x}{2}dx = \frac{1}{2\pi}\pi = \frac{1}{2}$$

$$f \sim \frac{1}{\pi} + \frac{1}{2}\sin x - \sum_{n=2}^{\infty}\frac{1+(-1)^n}{n}\cos nx$$