11.3.28.

$$f(x) = \sin x \ , \ 0 < x < \pi$$

a)

Fourierserien är på formen: $\dfrac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx$.

$$a_n = \frac{2}{\pi}\int_0^{\pi} \sin x \cos nx dx = \frac{1}{\pi}\int_0^{\pi} (\sin(n+1)x - \sin(n-1)x) dx =$$

$$= [n \neq 1] = \frac{1}{\pi}\left[\frac{-\cos(n+1)x}{n+1} + \frac{\cos(n-1)x}{n-1}\right]_0^{\pi} =$$

$$= \frac{1}{\pi}\left\{\frac{1-\cos(n+1)\pi}{n+1} - \frac{1-\cos(n-1)\pi}{n-1}\right\} =$$

$$= \frac{1}{\pi}\left\{\frac{1-(-1)^{n+1}}{n+1} - \frac{1-(-1)^{n+1}}{n-1}\right\} =$$

$$= \frac{1-(-1)^{n+1}}{\pi}\left\{\frac{1}{n+1} - \frac{1}{n-1}\right\} = -2\frac{1+(-1)^{n}}{\pi(n^{2}-1)}$$

$$a_1 = \frac{1}{\pi}\int_0^{\pi}\sin 2x\,dx = 0 \quad , \quad a_0 = \frac{4}{\pi}$$

$$f \sim \frac{2}{\pi} - \sum_{n=2}^{\infty} 2\,\frac{1+(-1)^n}{\pi(n^2-1)}\cos nx = \frac{2}{\pi} - \sum_{m=1}^{\infty} \frac{4}{\pi(4m^2-1)}\cos 2mx$$

b)

Fourierserien är på formen: $\sum_{n=1}^{\infty} b_n \sin nx$.

Den givna funktionen är sin egen Fourierserie .