11.3.42.

$$m\ddot{x} + kx = f(t) = \begin{cases} t, & 0 < t < \frac{1}{2} \\ 1 - t, & \frac{1}{2} \le t < 1 \end{cases}, \quad f(t+1) = f(t).$$

$$m = \frac{1}{4}, \quad k = 12.$$

$$\ddot{x} + 48x = 4f(t)$$

Fourierutveckla f.

f är en jämn funktion.

$$a_n = \frac{2}{1}\int_0^{\frac{1}{2}} 4t\cos\frac{n\pi t}{\frac{1}{2}}\,dt = 4\int_0^{\frac{1}{2}} 2t\cos 2n\pi t\,dt =$$

$$= \left[\text{partiell integration}\right] = 4\left\{ \left[t\frac{\sin 2n\pi t}{n\pi}\right]_0^{\frac{1}{2}} - \int_0^{\frac{1}{2}} 1\frac{\sin 2n\pi t}{n\pi}\,dt \right\} =$$

$$= 4\left[\frac{\cos 2n\pi t}{2(n\pi)^2}\right]_0^{\frac{1}{2}} = 2\,\frac{\cos n\pi - 1}{(n\pi)^2}$$

$$a_0 = \frac{2}{1}\int_0^{\frac{1}{2}} 4t\,dt = 4\left[t^2\right]_0^{\frac{1}{2}} = 1$$

$$f \sim \frac{2}{2} + \sum_{n=2}^{\infty} 4\frac{\cos n\pi - 1}{(n\pi)^2}\cos 2n\pi t$$

Vi ansätter $x_p = \frac{A_0}{2} + \sum_{n=2}^{\infty} A_n \cos 2n\pi t$.

$$\ddot{x}_p = \sum_{n=2}^{\infty} -4n^2\pi^2 A_n \cos 2n\pi t$$

Insättning i den givna diff.ekv. ger :

$$\sum_{n=2}^{\infty} -4n^2\pi^2 A_n \cos 2n\pi t + 48\left\{\frac{A_0}{2} + \sum_{n=2}^{\infty} A_n \cos 2n\pi t\right\} =$$

$$= 1 + \sum_{n=2}^{\infty} 4\,\frac{\cos n\pi - 1}{(n\pi)^2}\cos 2n\pi t$$

$$48\,\frac{A_0}{2} - 1 + \sum_{n=2}^{\infty}\left\{-4n^2\pi^2 A_n + 48A_n - 4\,\frac{\cos n\pi - 1}{(n\pi)^2}\right\}\cos 2n\pi t = 0$$

$$\frac{A_0}{2} = \frac{1}{48}\quad,\quad A_n = \frac{\cos n\pi - 1}{(n\pi)^2(12 - n^2\pi^2)}$$

$$x_p = \frac{1}{48} + \sum_{n=2}^{\infty}\frac{\cos n\pi - 1}{(n\pi)^2(12 - n^2\pi^2)}\cos 2n\pi t$$