12.1.11.

Variabelseparation.

$$a^2 \frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2} \;.$$ Vågekvationen.

Ansats: $u(x,t) = X(x)T(t).$

$$a^2 X''(x)T(t) = X(x)T''(t)$$

$$\frac{X''(x)}{X(x)} = \frac{T''(t)}{a^2 T(t)} = \text{konstant} = \lambda$$

Ett system av okopplade ODE erhålles.

$$X''(x) - \lambda X(x) = 0$$

$$T''(t) - \lambda a^2 T(t) = 0$$

Linjära med konstanta koefficienter.

Tre olika fall : $\lambda > 0,\ \lambda = 0,\ \lambda < 0.$

$$\lambda > 0,\ \lambda = \mu^2,\ \ \mu \quad R.$$

$$X''(x) - \mu^2 X(x) = 0$$

Lösningarna ges av $X(x) = A_1 e^{\mu x} + B_1 e^{-\mu x}$.

Motsvarande för "T-ekvationen" ger: $\mathrm{T(t)} = \mathrm{C}_1 e^{a\mu t} + D_1 e^{-a\mu t}$.

$$u(x,t) = (A_1 e^{\mu x} + B_1 e^{-\mu x})(\mathrm{C}_1 e^{a\mu t} + D_1 e^{-a\mu t})$$

$$\lambda = 0$$

$$X''(x) = 0$$

$$X(x) = A_2 x + B_2$$

$$T(t) = C_2 t + D_2$$

$$u(x,t) = (A_2 x + B_2)(C_2 t + D_2)$$

$$\lambda < 0,\ \lambda = -\mu^2,\ \ \mu \quad R.$$

$$X''(x) + \mu^2 X(x) = 0$$

$$X(x) = A_3 \cos \mu x + B_3 \sin \mu x$$

$$T(t) = C_3 \cos a\mu t + D_3 \sin a\mu t$$

$$u(x,t) = (A_3 \cos \mu x + B_3 \sin \mu x)(C_3 \cos a\mu t + D_3 \sin a\mu t)$$