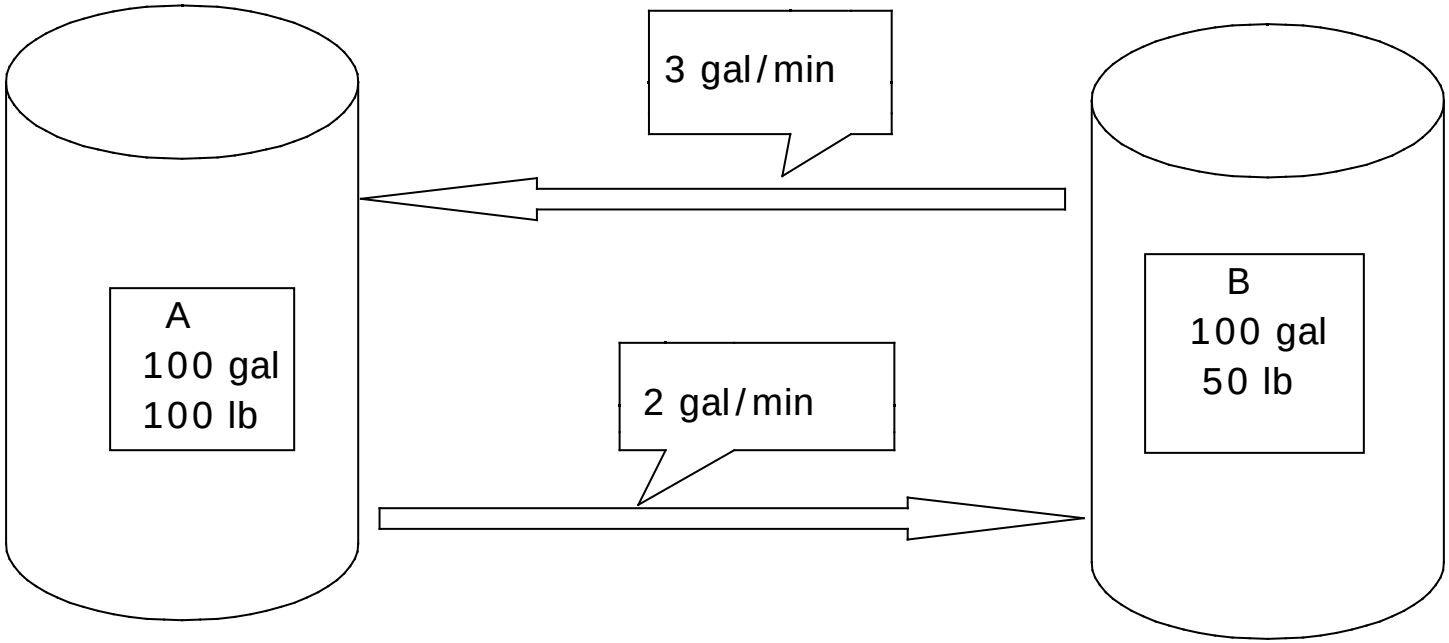
Z.C.3.3.7.



a)

$$\frac{dx_1}{dt}\ lb/\min = \ 3\ gal/\min * \frac{x_2(t)}{100-t}\ lb/gal - 2\ gal/\min\ \frac{x_1(t)}{100+t}\ lb/gal$$

$$\frac{dx_2}{dt}\ lb/\min = \ -3\ gal/\min * \frac{x_2(t)}{100-t}\ lb/gal + 2\ gal/\min\ \frac{x_1(t)}{100+t}\ lb/gal$$

$$x_1(0) = 100,\ \ x_2(0) = 50$$

$$\frac{dx_1}{dt} = \ 3\frac{x_2(t)}{100-t} - 2\frac{x_1(t)}{100+t}$$

$$\frac{dx_2}{dt} = \ -3\frac{x_2(t)}{100-t} + 2\frac{x_1(t)}{100+t}$$

$$x_1(0) = 100,\ \ x_2(0) = 50$$

b)

$$\frac{dx_1}{dt} + \frac{dx_2}{dt} = \frac{d(x_1 + x_2)}{dt} = 0$$

$$x_1 + x_2 = \text{konstant} = x_1(0) + x_2(0) = 100 + 50 = 150$$

Systemet är slutet.

Eliminera x_1: $x_1 = 150 - x_2$.

$$\frac{dx_2}{dt} = -3\frac{x_2(t)}{100 - t} + 2\frac{150 - x_2(t)}{100 + t}$$

$$\frac{dx_2}{dt} + \left(\frac{3}{100 - t} + \frac{2}{100 + t}\right)x_2(t) = \frac{300}{100 + t}$$

Linjär diff.ekv. av 1:a ordningen.

Bestäm en integrerande faktor:

$$\exp\{\int(\frac{3}{100-t}+\frac{2}{100+t})dt\}=(100+t)^2(100-t)^{-3}.$$

$$\frac{d}{dt}\{x_2(t)(100+t)^2(100-t)^{-3}\}= \quad 300(100+t)(100-t)^{-3}$$

$$x_2(t)(100+t)^2(100-t)^{-3}=\{\text{Partiell integration.}\}=$$

$$=300\{(100+t)\frac{(100-t)^{-2}}{+2}-\int\frac{(100-t)^{-2}}{+2}dt\}=$$

$$= 300\{(100+t)\frac{(100-t)^{-2}}{+2} - \frac{(100-t)^{-1}}{+2} + C\}$$

$$x_2(t) = \frac{300}{(100+t)^2}\{\frac{(100+t)(100-t)}{+2} - \frac{(100-t)^2}{+2} + C(100-t)^3\}$$

$$x_2(t) = \frac{300}{(100+t)^2}\left\{C(100-t)^3 + t(100-t)\right\}$$

$$x_2(0) = 50 \qquad 50\frac{(100)^2}{300} = C(100)^3$$

$$C = \frac{50}{300*100} = \frac{1}{600}$$

$$x_2(t) = \frac{300}{(100+t)^2}\left\{\frac{(100-t)^3}{600} + t(100-t)\right\}$$

$$x_2(30) = \frac{300}{(100+30)^2}\left\{\frac{(100-30)^3}{600} + 30(100-30)\right\} =$$

$$= \frac{3}{(13)^2}\left\{\frac{(7)^3 5}{3} + 2100\right\} = \frac{1}{(13)^2}\{1715 + 6300\} = \frac{8015}{169} \approx 47.4 \ \ lbs$$