

7.3.30.

$$y'' - 2y' + 5y = 1 + t, \quad y(0) = 0, \quad y'(0) = 4$$

$$s^2 Y(s) - sy(0) - y'(0) - 2(sY(s) - y(0)) + 5Y(s) = \frac{1}{s} + \frac{1}{s^2}$$

$$s^2 Y(s) - 4 - 2sY(s) + 5Y(s) = \frac{1}{s} + \frac{1}{s^2}$$

$$Y(s) = \frac{4}{s^2 - 2s + 5} + \frac{1}{s(s^2 - 2s + 5)} + \frac{1}{s^2(s^2 - 2s + 5)}$$

$$Y(s) = \frac{4s^2 + s + 1}{s^2(s^2 - 2s + 5)} = \frac{A}{s^2} + \frac{B}{s} + \frac{Cs + D}{s^2 - 2s + 5}$$

$$\text{Hpl ger: } A = \frac{1}{5}.$$

$$4s^2 + s + 1 = \frac{1}{5}(s^2 - 2s + 5) + Bs(s^2 - 2s + 5) + s^2(Cs + D)$$

$$s^3: 0 = B + C$$

$$s^2: 4 = \frac{1}{5} - 2B + D$$

$$s: 1 = \frac{-2}{5} + 5B$$

$$C = -B$$

$$D = 2B + \frac{19}{5}$$

$$B = \frac{7}{25}$$

$$C = -\frac{7}{25}$$

$$D = \frac{14}{25} + \frac{95}{25} = \frac{109}{25}$$

$$B = \frac{7}{25}$$

$$Y(s) = \frac{1}{25} \left(\frac{5}{s^2} + \frac{7}{s} + \frac{-7s + 109}{s^2 - 2s + 5} \right)$$

$$Y(s) = \frac{1}{25} \left(\frac{5}{s^2} + \frac{7}{s} + \frac{-7(s-1) + 51 \cdot 2}{(s-1)^2 + 4} \right)$$

$$y(t) = \frac{1}{25} (5t + 7 - e^t (7 \cos 2t - 51 \sin 2t))$$