

7.6.12.

$$\begin{aligned}x' &= 4x - 2y + 2U(t-1) & x(0) &= 0 \\y' &= 3x - y + U(t-1) \quad , \quad y(0) = 1/2\end{aligned}$$

$$\begin{aligned}sX(s) - 0 &= 4X(s) - 2Y(s) + \frac{2}{s}e^{-s} \\sY(s) - 1/2 &= 3X(s) - Y(s) + \frac{1}{s}e^{-s}\end{aligned}$$

$$\begin{aligned}(s-4)X(s) + 2Y(s) &= \frac{2}{s}e^{-s} \\-3X(s) + (s+1)Y(s) &= 1/2 + \frac{1}{s}e^{-s}\end{aligned}$$

$$X(s) = \frac{\begin{vmatrix} \frac{2}{s}e^{-s} & 2 \\ 1/2 + \frac{1}{s}e^{-s} & s+1 \end{vmatrix}}{\begin{vmatrix} s-4 & 2 \\ -3 & s+1 \end{vmatrix}} = \frac{2e^{-s} + \frac{2}{s}e^{-s} - 1 - \frac{2}{s}e^{-s}}{s^2 - 3s + 2}$$

$$Y(s) = \frac{\begin{vmatrix} s-4 & \frac{2}{s}e^{-s} \\ -3 & 1/2 + \frac{1}{s}e^{-s} \end{vmatrix}}{\begin{vmatrix} s-4 & 2 \\ -3 & s+1 \end{vmatrix}} = \frac{\frac{s}{2} + e^{-s} - \frac{4}{s}e^{-s} - 2 + \frac{6}{s}e^{-s}}{s^2 - 3s + 2}$$

$$X(s) = \frac{2e^{-s} - 1}{(s - 1)(s - 2)}$$

$$Y(s) = \frac{(s - 4)/2}{(s - 1)(s - 2)} + e^{-s} \frac{2 + s}{s(s - 1)(s - 2)}$$

$$Y(s) = \frac{1}{2} \left( \frac{1}{(s - 1)} - \frac{2}{(s - 1)(s - 2)} \right) + e^{-s} \left( \frac{1}{s} - \frac{3}{(s - 1)} + \frac{2}{(s - 2)} \right)$$

$$x(t) = -\frac{-e^{+1t} + e^{+2t}}{-1 - (-2)} + 2U(t - 1)\{\frac{-e^{+1(t-1)} + e^{+2(t-1)}}{-1 - (-2)}\}$$

$$y(t) = \frac{1}{2}\{e^{+1t} - 2(\frac{-e^{+1t} + e^{+2t}}{-1 - (-2)})\} + U(t - 1)\{1 - 3e^{+1(t-1)} + 2e^{+2(t-1)}\}$$

$$x(t) = e^t - e^{+2t} + 2U(t - 1)\{e^{2(t-1)} - e^{(t-1)}\}$$

$$y(t) = \frac{1}{2}\{3e^t - 2e^{2t}\} + U(t - 1)\{1 - 3e^{(t-1)} + 2e^{2(t-1)}\}$$