8.2.10.

$$\mathbf{X}' = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} \mathbf{X} = \mathbf{A}\mathbf{X}$$

$$0 = \det(\mathbf{A} - \lambda \mathbf{I}) = \begin{vmatrix} 1-\lambda & 0 & 1 \\ 0 & 1-\lambda & 0 \\ 1 & 0 & 1-\lambda \end{vmatrix} =$$

$$= (1-\lambda)\left\{(1-\lambda)^2 - 1\right\} = (1-\lambda)(1-\lambda+1)(1-\lambda-1) =$$

$$= (1-\lambda)(2-\lambda)(-\lambda)$$

$$\lambda_1 = 0 \;,\; \lambda_2 = 1 \;,\; \lambda_3 = 2$$

Bestäm en egenvektor till varje egenvärde .

$\lambda_1 = 0$

Insättning i $(\mathbf{A} - \lambda\mathbf{I})\mathbf{v} = \mathbf{0}$ ger:

$$\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} \mathbf{v}_1 = \mathbf{0} \qquad \mathbf{v}_1 = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$$

$$\lambda_2 = 1$$

$$\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \mathbf{v}_2 = \mathbf{0} \qquad \mathbf{v}_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$\lambda_3 = 2$$

$$\begin{pmatrix} -1 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & -1 \end{pmatrix} \mathbf{v}_3 = \mathbf{0} \qquad \mathbf{v}_3 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

$$\mathbf{X} = c_1 \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} + c_2 \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} e^{t} + c_3 \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} e^{2t} =$$

$$= \begin{pmatrix} 1 & 0 & e^{2t} \\ 0 & e^{t} & 0 \\ -1 & 0 & e^{2t} \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix}$$

```
> ekv1:=diff(x(t),t)=x(t)+z(t):
> ekv2:=diff(y(t),t)=y(t):
> ekv3:=diff(z(t),t)=x(t)+z(t):

> dsolve({ekv1,ekv2,ekv3},{x(t),y(t),z(t)});

 {y(t) = exp(t) _C1,

 x(t) = 1/2 _C2 exp(2 t) + 1/2 _C2 +

1/2 _C3 exp(2 t) - 1/2 _C3,

 z(t) = 1/2 _C2 exp(2 t) - 1/2 _C2 +

1/2 _C3 exp(2 t) + 1/2 _C3}
```