8.2.36.

$$\begin{cases} \dfrac{dx}{dt} = 4x + 5y \\ \dfrac{dy}{dt} = -2x + 6y \end{cases} \qquad \begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 4 & 5 \\ -2 & 6 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

$$0 = \begin{vmatrix} 4-\lambda & 5 \\ -2 & 6-\lambda \end{vmatrix} = (4-\lambda)(6-\lambda) + 10 =$$

$$= \lambda^2 - 10\lambda + 34 = (\lambda - 5)^2 + 9$$

$$\lambda_{1,2} = 5 \pm 3i$$

Bestäm en komplex egenvektor.

Insättning i $(\mathbf{A}-\lambda\mathbf{I})\mathbf{v}=\mathbf{0}$ ger:

$$\begin{pmatrix} 4-5-3i & 5 \\ -2 & 6-5-3i \end{pmatrix}\mathbf{v}_1=\mathbf{0}$$

$$\begin{pmatrix} -1-3i & 5 \\ -2 & 1-3i \end{pmatrix}\mathbf{v}_1=\mathbf{0} \quad \mathbf{v}_1=\begin{pmatrix} 1-3i \\ 2 \end{pmatrix}$$

$$\mathbf{Z}=e^{(5+3i)t}\begin{pmatrix} 1-3i \\ 2 \end{pmatrix}=e^{5t}(\cos 3t+i\sin 3t)\left\{\begin{pmatrix} 1 \\ 2 \end{pmatrix}+i\begin{pmatrix} -3 \\ 0 \end{pmatrix}\right\}$$

$$\begin{cases} \mathbf{X}_1 = \operatorname{Re}(\mathbf{Z}) = e^{5t}\left(\begin{pmatrix}1\\2\end{pmatrix}\cos 3t + \begin{pmatrix}3\\0\end{pmatrix}\sin 3t\right) \\ \mathbf{X}_2 = \operatorname{Im}(\mathbf{Z}) = e^{5t}\left(\begin{pmatrix}-3\\0\end{pmatrix}\cos 3t + \begin{pmatrix}1\\2\end{pmatrix}\sin 3t\right) \end{cases}$$

$$\begin{cases} \mathbf{X}_1 = e^{5t}\begin{pmatrix}\cos 3t + 3\sin 3t\\ 2\cos 3t\end{pmatrix} \\ \mathbf{X}_2 = e^{5t}\begin{pmatrix}\sin 3t - 3\cos 3t\\ 2\sin 3t\end{pmatrix} \end{cases}$$

$$\mathbf{X} = c_1 e^{5t}\begin{pmatrix}\cos 3t + 3\sin 3t\\ 2\cos 3t\end{pmatrix} + c_2 e^{5t}\begin{pmatrix}\sin 3t - 3\cos 3t\\ 2\sin 3t\end{pmatrix}$$