8.2.44.

$$\mathbf{X}' = \begin{pmatrix} 2 & 4 & 4 \\ -1 & -2 & 0 \\ -1 & 0 & -2 \end{pmatrix} \mathbf{X}$$

$$0 = \begin{vmatrix} 2-\lambda & 4 & 4 \\ -1 & -2-\lambda & 0 \\ -1 & 0 & -2-\lambda \end{vmatrix} =$$

$$= (2-\lambda)(2+\lambda)^2 - 4(2+\lambda) - 4(2+\lambda) =$$

$$= (2+\lambda)\left\{4-\lambda^2-8\right\} = -(2+\lambda)(4+\lambda^2)$$

$$\lambda_1 = -2 \;,\; \lambda_{2,3} = \pm 2i$$

Insättning i $(\mathbf{A}-\lambda\mathbf{I})\mathbf{v}=\mathbf{0}$ ger:

$$\lambda_1=-2$$

$$\begin{pmatrix}4 & 4 & 4\\ -1 & 0 & 0\\ -1 & 0 & 0\end{pmatrix}\mathbf{v}_1=\mathbf{0}\ ,\quad \mathbf{v}_1=\begin{pmatrix}0\\ 1\\ -1\end{pmatrix}$$

$$\mathbf{X}_1=\begin{pmatrix}0\\ 1\\ -1\end{pmatrix}e^{-2t}=\begin{pmatrix}0\\ e^{-2t}\\ -e^{-2t}\end{pmatrix}$$

Bestäm en komplex egenvektor.

Insättning i $(\mathbf{A}-\lambda\mathbf{I})\mathbf{v}=\mathbf{0}$ ger:

$$\begin{pmatrix} 2-2i & 4 & 4 \\ -1 & -2-2i & 0 \\ -1 & 0 & -2-2i \end{pmatrix}\mathbf{v}_2=\mathbf{0} \qquad \mathbf{v}_2=\begin{pmatrix} 2+2i \\ -1 \\ -1 \end{pmatrix}$$

$$\mathbf{Z}=e^{2it}\begin{pmatrix} 2+2i \\ -1 \\ -1 \end{pmatrix}=(\cos 2t+i\sin 2t)\left\{\begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix}+i\begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix}\right\}$$

$$\begin{cases} \mathbf{X}_2 = \operatorname{Re}(\mathbf{Z}) = \left(\begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix} \cos 2t - \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix} \sin 2t \right) \\ \mathbf{X}_3 = \operatorname{Im}(\mathbf{Z}) = \left(\begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix} \cos 2t + \begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix} \sin 2t \right) \end{cases}$$

$$\mathbf{X}_2 = \begin{pmatrix} 2\cos 2t - 2\sin 2t \\ -\cos 2t \\ -\cos 2t \end{pmatrix}, \quad \mathbf{X}_3 = \begin{pmatrix} 2\cos 2t + 2\sin 2t \\ -\sin 2t \\ -\sin 2t \end{pmatrix}$$

$$\mathbf{X} = c_1 \begin{pmatrix} 0 \\ e^{-2t} \\ -e^{-2t} \end{pmatrix} + c_2 \begin{pmatrix} 2\cos 2t - 2\sin 2t \\ -\cos 2t \\ -\cos 2t \end{pmatrix} + c_3 \begin{pmatrix} 2\cos 2t + 2\sin 2t \\ -\sin 2t \\ -\sin 2t \end{pmatrix}$$