8.3.13.

$$\mathbf{X}' = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \mathbf{X} + \begin{pmatrix} \cos t \\ \sin t \end{pmatrix} e^t$$

Bestäm en fundamentalmatris , $\Phi(t)$.

$$\mathbf{X}' = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \mathbf{X}$$

$$0 = \begin{vmatrix} 1-\lambda & -1 \\ 1 & 1-\lambda \end{vmatrix} = (1-\lambda)^2 + 1$$

$$\lambda_{1,2} = 1 \pm i$$

Bestäm en komplex egenvektor.

Insättning i $(\mathbf{A} - \lambda \mathbf{I})\mathbf{v} = \mathbf{0}$ ger:

$$\begin{pmatrix} 1-(1+i) & -1 \\ 1 & 1-(1+i) \end{pmatrix} \mathbf{v}_1 = \mathbf{0}$$

$$\begin{pmatrix} -i & -1 \\ 1 & -i \end{pmatrix} \mathbf{v}_1 = \mathbf{0} \;, \quad \mathbf{v}_1 = \begin{pmatrix} 1 \\ -i \end{pmatrix}$$

$$\mathbf{Z} = e^{(1+i)t} \begin{pmatrix} 1 \\ -i \end{pmatrix} = e^{t}(\cos t + i \sin t) \left\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix} + i \begin{pmatrix} 0 \\ -1 \end{pmatrix} \right\}$$

$$\begin{cases} \mathbf{X}_1 = \mathrm{Re}(\mathbf{Z}) = e^t \left(\begin{pmatrix} 1 \\ 0 \end{pmatrix} \cos t + \begin{pmatrix} 0 \\ 1 \end{pmatrix} \sin t \right) \\ \mathbf{X}_2 = \mathrm{Im}(\mathbf{Z}) = e^t \left(\begin{pmatrix} 0 \\ -1 \end{pmatrix} \cos t + \begin{pmatrix} 1 \\ 0 \end{pmatrix} \sin t \right) \end{cases}$$

$$\mathbf{X}_1 = e^t \begin{pmatrix} \cos t \\ \sin t \end{pmatrix}, \quad \mathbf{X}_2 = e^t \begin{pmatrix} \sin t \\ -\cos t \end{pmatrix}$$

En fundamentalmatris $\Phi(t) = e^t \begin{pmatrix} \sin t & \cos t \\ -\cos t & \sin t \end{pmatrix}$.

$$\mathbf{X_p} = \Phi(t)\mathbf{U} = \Phi(t) \int \Phi^{-1}(t) \mathbf{F}(t) \mathbf{dt}$$

$$\Phi^{-1}(t) = e^{-t}\begin{pmatrix} \sin t & -\cos t \\ \cos t & \sin t \end{pmatrix}$$

$$\mathbf{U} = \int e^{-t}\begin{pmatrix} \sin t & -\cos t \\ \cos t & \sin t \end{pmatrix}\begin{pmatrix} \cos t \\ \sin t \end{pmatrix} e^{t}\,\mathbf{dt} =$$

$$= \int \begin{pmatrix} 0 \\ 1 \end{pmatrix}\mathbf{dt} = \begin{pmatrix} 0 \\ t \end{pmatrix}$$

$$\mathbf{X}_{\mathbf{p}} = e^{t}\begin{pmatrix} \sin t & \cos t \\ -\cos t & \sin t \end{pmatrix}\begin{pmatrix} 0 \\ t \end{pmatrix} = e^{t}\begin{pmatrix} t\cos t \\ t\sin t \end{pmatrix}$$

$$\mathbf{X} = \Phi(t)\mathbf{C} + \Phi(t)\mathbf{U} = e^{t}\begin{pmatrix} \sin t & \cos t \\ -\cos t & \sin t \end{pmatrix}\mathbf{C} + te^{t}\begin{pmatrix} \cos t \\ \sin t \end{pmatrix}$$