KUNGL TEKNISKA HÖGSKOLAN

Matematiska institutionen
Avdelningen för Optimeringslära och systemteori

Homework 1: 5B5765: Robust Control Theory Fall 2003
Deadline: October 6, 2003.

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1. Prove the following result

(i) for every state feedback F and any invertible input transformation M we have

$$(A, B) \text{ stabilizable} \Longleftrightarrow (A + BF, BM) \text{ is stabilizable}$$

(ii) for every output feedback L and any invertible output transformation M we have

$$(C, A) \text{ detectable} \Longleftrightarrow (MC, A + LC) \text{ is detectable}$$

2. A generalized eigenvalue problem for the matrix pair (M, N) is the problem of finding a generalized eigenvalue λ and a generalized eigenvector x s.t.

$$Mx = \lambda Nx$$

(a) Show how to compute all the invariant zeros of a system

$$G(s) = \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right]$$

using generalized eigenvalue problems.

(b) Use the Matlab command eig(M,N) to compute the invariant zeros of

$$G(s) = \left[\begin{array}{cc|cc} 1 & 2 & 2 & 2 \\ 1 & 0 & 3 & 4 \\ \hline 0 & 1 & 1 & 2 \\ 1 & 0 & 2 & 0 \end{array}\right]$$

3. Define the sensitivity function and the complementary sensitivity function as

$$S = (I - PK)^{-1},$$
$$T = (I - PK)^{-1}PK,$$

respectively.

(a) Prove that $S - T = I$

(b) Suppose P and K are SISO systems (single input single output). Let z_i and p_i denote the zeros respectively the poles of $P(s)K(s)$. Use Theorem 1 in the lecture notes on internal stability to derive interpolation conditions on S and T, i.e. constants s_{zi} such that $S(z_i) = s_{zi}$ e.t.c.

4. Prove that

$$\| \begin{bmatrix} G & I \end{bmatrix} \|^2_{\mathbf{H}_\infty} = \|G\|^2_{\mathbf{H}_\infty} + 1$$

5. Consider a transfer function with realization

$$G(s) = \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array} \right]$$

Show that if λ is an eigenvalue of A then either λ is a pole of $G(s)$ or it corresponds to an unobservable or uncontrollable mode.
This implies that if the realization is stabilizable and detectable then A Hurwitz $\Longleftrightarrow$ *$G(s)$ stable (unstable pole-zero cancelations are impossible).*