KUNGL TEKNISKA HÖGSKOLAN

Matematiska institutionen
Avdelningen för Optimeringslära och systemteori

Homework 2: 5B5765: Robust Control Theory Fall 2003
Deadline: October 20, 2003.

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1. Prove the submultiplicative property of the induced norm, i.e. if $G \in \mathcal{L}(\mathcal{U}, \mathcal{V})$ and $H \in \mathcal{L}(\mathcal{V}, \mathcal{W})$ then

$$\|HG\|_{\mathcal{U}\to\mathcal{W}} \leq \|H\|_{\mathcal{V}\to\mathcal{W}} \|G\|_{\mathcal{U}\to\mathcal{V}}$$

2. Consider $G(i\omega) = \dfrac{2}{(i\omega+1)(i\omega-1)} \in \mathbf{L}_2(j\mathbf{R})$. Determine the time domain representation of this operator, i.e. consider the interpretation $G \in \mathcal{L}(\mathbf{L}_2(\mathbf{R}))$ and determine the convolution representation.

3. Let $G(s) = C(sI - A)^{-1}B \in \mathbf{RH}_\infty$, i.e. A is Hurwitz. Consider the time domain represenation $G \in \mathcal{L}(\mathbf{L}_2(\mathbf{R}))$.

(a) Determine the convolution representation of G

(b) Determine the convolution representation of the adjoint operator $G^* \in \mathcal{L}(\mathbf{L}_2(\mathbf{R}))$. Recall that the adjoint satisfies

$$\langle Gu, v\rangle = \langle u, G^*v\rangle\,, \qquad \forall u, v \in \mathbf{L}_2(\mathbf{R})$$

(c) Determine the convolution representation of the adjoint operator $G^* \in \mathcal{L}(\mathbf{L}_2[0,\infty))$.

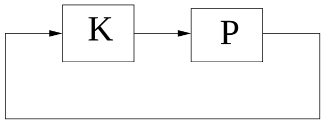
(d) Determine the convolution representation of the adjoint operator $G^* \in \mathcal{L}(\mathbf{L}_2(-\infty, 0])$.

(e) Let $G(s) = C(sI - A)^{-1}B \in \mathbf{RL}_\infty$. Determine a state space realization for the adjoint operator G^*. When is G self adjoint?

4. (From Zhou) Let $M = \begin{bmatrix} 0 & M_{12} \\ M_{21} & 0 \end{bmatrix}$ be a complex matrix and let $\Delta = \begin{bmatrix} \Delta_1 & 0 \\ 0 & \Delta_2 \end{bmatrix}$. Show that

$$\mu_{\mathbf{\Delta}}(M) = \sqrt{\overline{\sigma}(M_{12})\overline{\sigma}(M_{21})}$$

5. (From Zhou) Consider the feedback interconnection



Suppose that all transfer functions are SISO and that

$$P = P_0(1 + W_1\Delta_1) + W_2\Delta_2$$

where the W_k and Δ_k are stable and $\|\Delta_k\|_{\mathbf{H}_\infty} \leq 1,\ k = 1, 2$. Suppose also that K internally stabilizes P_0. Show that a necessary and sufficient condition for robust stability is that

$$\| |W_1 T| + |W_2 K S| \|_\infty < 1$$

where

$$S = \frac{1}{1 + P_0 K}, \qquad T = \frac{P_0 K}{1 + P_0 K}.$$