Kungl Tekniska Högskolan

Matematiska institutionen
Avdelningen för Optimeringslära och systemteori

Homework 3: 5B5765: Robust Control Theory Fall 2003
Deadline: November 3, 2003.

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1. Let $G(s) = \left[\begin{array}{c|c} A & B \\ \hline C & 0 \end{array}\right] \in \mathbf{RH}_\infty$ be a minimal realization. Use the KYP lemma to show that the following are equivalent.

(i) $\|G\|_{\mathbf{H}_\infty} < \gamma$

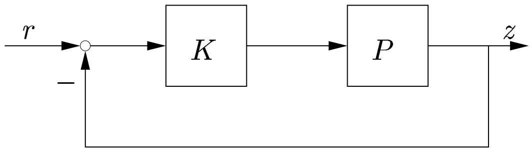
(ii) $H = \begin{bmatrix} A & -\frac{1}{\gamma^2}BB^T \\ C^TC & -A^T \end{bmatrix}$ has no eigenvalues on the imaginary axis.

(iii) H belongs to dom(Ric) and Ric$(H) \geq 0$.

2. Let G have the state space realization in problem 1. If $w_i(t) = \delta(t)e_i$, where $\delta(t)$ is the dirac distribution and e_i is the i^{th} unit vector, then

$$\sum_{i=1}^{m} \|Gw_i\|_2^2 = \|G\|_{\mathbf{H}_2}^2$$

3. (Adopted from Dullerud/Paganini) Consider the system below. Suppose $P \in \mathbf{RH}_\infty$.



(a) Show that the Youla parametrization simplifies to

$$K = Q(I - PQ)^{-1}, \qquad \det(I - P(\infty)Q(\infty)) \neq 0$$

(b) Let P, K be SISO. Find a parametrization of all controllers that give perfect asymptotic tracking of a step function $r(t) = r_0\theta(t)$, where $\theta(t)$ is the unit step function.

4. (From Zhou) Let $G(s) = \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right]$, where (A, B) is stabilizable and (C, A) is detectable. Suppose $G(j\omega)^* G(j\omega) > \epsilon I$, $\forall \omega \in \mathbf{R}$. Let

$$X = \mathrm{Ric}\left(\begin{bmatrix} A - BR^{-1}D^T C & -BR^{-1}B^T \\ -C^T(I - DR^{-1}D^T)C & -(A - BR^{-1}D^T C)^T \end{bmatrix}\right)$$

where $R = D^T D$.

(a) Prove that $W^* W = G^* G$, where

$$W = \left[\begin{array}{c|c} A & B \\ \hline R^{-1/2}(D^T C + B^T X) & R^{1/2} \end{array}\right]$$

(b) Prove that $W^{-1} \in \mathbf{RH}_\infty$