Robust Control Theory

5b5765

Ulf Jönsson

Optimization and Systems Theory

Royal Institute of Technology

Sweden

Outline

- Course requirements
- Course material
- Some highlights of the course
- Preliminary schedule

Course Requirements

5B5765 is a graduate course on robust control theory. It is mainly intended for graduate students in control and systems theory. It is recommended that the students have taken 5B1742 (Matematisk systemteori) or some similar course.

- The course will give 6 credits
- There will be 1-2 lectures each week
- There will be six homework sets and a final exam
- It is possible to give guest seminars.

Course Material

- Lecture notes will be available at http://www.math.kth.se/ uj/5B5765/notes.html. These notes cover most of the material discussed during lectures but is not sufficient. We recommend the books
  - Essentials of Robust Control, by Zhou and Doyle
  - A course in Robust Control Theory, by Dullerud and Paganini
- In each lecture we also supply references to key papers and monographs.
- There are several useful homepages for robust control courses. We list a few
  - Department of Automatic Control, LTH: Basics of Robust Control

http://www.control.lth.se/ ghulchak/lectures/robust/robust.html

- LIDS MIT. Linear Multivariable Systems by A. Megretski http://web.mit.edu/6.245/www/
- The homepage of Carsten Schererer at http://www.ocp.tudelft.nl/sr/personal/Scherer/

- Robust control as a subject and research direction is huge. In the course we cover several important directions. To learn more about this subject at KTH we recomend
  - 5B5742 Topics in Control and Systems Theory
    * L_1 optimal control
    * Sampled data control
    * Gap metric topology, e.t.c.
  - Multivariable Feedback Control, Process Control, KTH.
  - Interpolation methods: See research group of A. Lindquist.

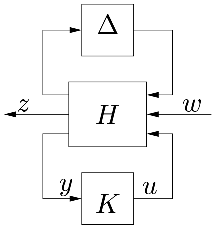
Highlights of the Course

Uncertainty appear naturally when modeling physical systems. We consider two types of uncertainty

1. Model uncertainty due to insufficient precision or approximation during the modeling phase
   - System identification only yields approximate models. Parameters cannot be determined exactly and the chosen model order may be to low
   - Delays may be ignored or uncertain
   - Nonlinearities may be ignored or may not be exactly known
   - Time-varying components may be approximated as time invariant or the time-variation may not be precisely known
2. Disturbance signals that may appear due to environmental

imperfections such as noise in amplifiers, temperature variations e.t.c. In the course we mainly consider
   - Stochastic signals ($\mathbf{H}_2$ optimization)
   - Deterministic signals with bounded energy ($\mathbf{H}_\infty$ optimization)

The starting point of the course is models of the form



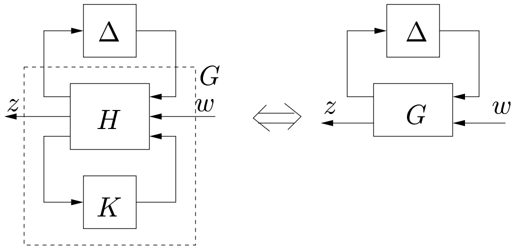
- Δ represents a model uncertainty. Mathematically we treat Δ as a bounded operator (You will learn what this means). Some common examples are uncertain parameters, dynamic

uncertainty (unknown transfer function), sector bounded nonlinearity.

- H represents the nominal dynamics. This is a finite dimensional transfer function.
- K is the controller, which also is a finite dimensional transfer function in this course.
- z regulated output
- y sensor output
- w exogenous input (disturbances)
- u control input

Systems Analysis

For given controller K we replace the feedback interconnection of H and K by a closed loop transfer function G.



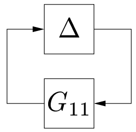
Some topics that will be covered in the course

- G is obtained as the lower linear fractional transformation (LFT) of H with respect to K. With appropriate block partitioning of

H we have

$$G = H_{11} + H_{12}K(I - H_{22}K)^{-1}H_{21}$$

- It requires care to ensure that the LFT is well posed and stable in an appropriate way. This is discussed in the section on internal stability.
- Let $G = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix}$ is a block partition consistent with the four signals. Suppose that G is stable
  - Stability of the closed loop follows if the following system is stable



Several examples appear

- ∗ If Δ is an unknown stable linear multivariable transfer function then we use the $\mathbf{H}_\infty$ norm to prove stability.
- ∗ If Δ in addition has structure (e.g., block diagonal) then we use the structured singular value to prove stability.
- ∗ If Δ is a more general uncertainty then we use integral quadratic constraints (IQC) to prove stability.

- If the interconnection of G_{11} and Δ is stable then we can consider performance analysis for the closed loop system (an upper LFT)

$$G_{cl} = G_{22} + G_{21}\Delta(I - G_{11}\Delta)^{-1}G_{12}$$

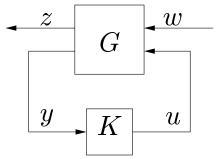
Examples are

- the energy gain (induced $\mathbf{L}_2$-norm which is equal to the $\mathbf{H}_\infty$ norm for linear time invariant systems).
- $\mathbf{H}_2$-norm (see below)

The methods used for performance analysis are based on the

- main loop theorem (LTI systems)
- the S-procedure (more general systems)

Synthesis



Some topics covered in the course

- The Youla parametrization, which is the set of all proper linear stabilizing controllers.
- $\mathbf{H}_2$ optimal control, which gives the controller that minimizes the output variance when the input is stochastic white noise signal. This is a special case of LQG. The modern approach is used to derive the result, where the separation between control and estimation is proven using the Youla parameterization.
- $\mathbf{H}_\infty$ optimal control, which gives the controller that minimizes

energy gain from input to output. If the minimal gain is γ then any system with gain less than $1/\gamma$ can be connected between the input and the output.

- Synthesis based on linear linear matrix inequalities. With a few clever ideas it is possible to derive a large number of synthesis results. Special cases are $\mathbf{H}_2$ and $\mathbf{H}_\infty$ optimal controllers. It is also possible to consider various multi-criteria objectives in the design.

Topics

1. Introduction
2. Linear multivariable systems
   - controllability and observability
   - stabilizability and detectability
   - observer based controllers
   - operations on transfer functions
   - poles and zeros of multivariable systems
3. Internal stability
4. Elementary operator theory
   - Function spaces representing signals
   - Operators representing systems

. The Structured singular value
   - Linear fractional transformation
   - Robustness of uncertain LTI systems

6. Kalman-Yakubovich-Popov lemma
7. The Youla parameterization
8. $\mathbf{H}_2$ optimal control
9. $\mathbf{H}_\infty$ optimal control
10. Integral quadratic constraints
11. Linear matrix inequalities
    - Basic concepts
    - Analysis results
    - Synthesis results