

# Sampled-Data Systems

## Lecture #1

Hisaya Fujioka

fujioka@i.kyoto-u.ac.jp, math.kth.se

Graduate School of Informatics, Kyoto University

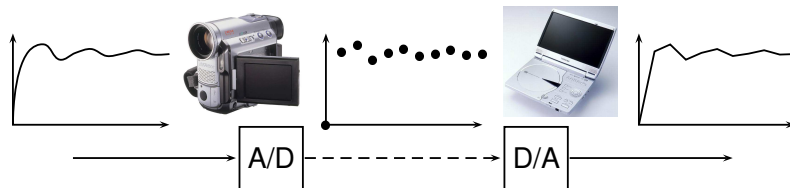
## Outline

- Sampled-Data Systems: What and Why ?
- Design of Sampled-Data Systems
  - Conventional Methods
  - Goal of This Course
- Experimental Examples
  - Hard Disk Drive
  - Ultrasonic Motor

## Sampled-Data Systems: What and Why ?

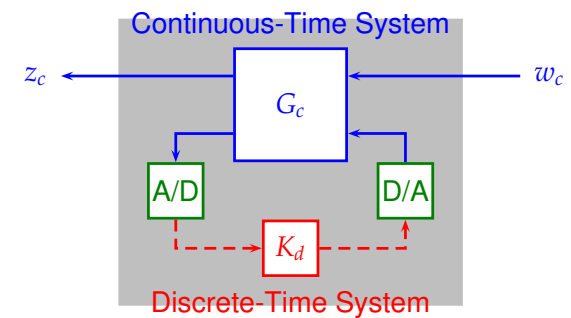
### Sampled-Data Systems

- **What:** Interconnection of continuous-time and discrete-time systems through A/D (analog-to-digital) and D/A (digital-to-analog) converters
- **Why:** Development of digital technology  
→ Most of engineering systems are SDSs



- **Attention:** Signals of interest are usually **analog signals**

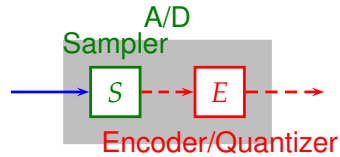
## Framework of Sampled-Data Systems



- Digital Control Systems
- Digital Signal Processing
- Digital Communication Systems
- ⋮

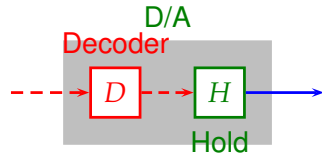
## A/D and D/A Converters

### A/D Converter



- **Sample:** Discretization of domain of signal
- **Encode/Quantize:** Discretization of range of signal  
 ← Digital computer can handle only finite number of values

### D/A Converter



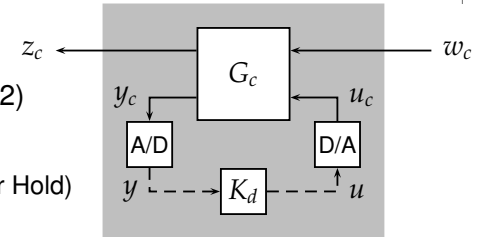
- **Decode**
- **Hold**

Sampled-Data Systems – p.5/22

## Assumptions

We assume

- $G_c$ : FDLTI
- $K_d$ : FDLTI (Lecture #2)
- A/D: **Ideal Sampler**
- D/A: **ZOH** (Zero Order Hold)



**Ideal Sampler** periodic sampling without encoding

$$S : y_c \mapsto y, \quad y[k] = y_c(kh) \quad h > 0: \text{ sampling period}$$

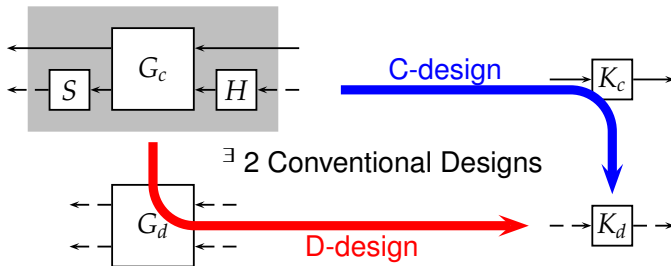
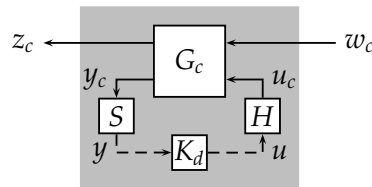
**ZOH** constant output through sampling interval

$$H : u \mapsto u_c, \quad u_c(kh + \theta) = u[k], \quad \theta \in [0, h)$$

Sampled-Data Systems – p.6/22

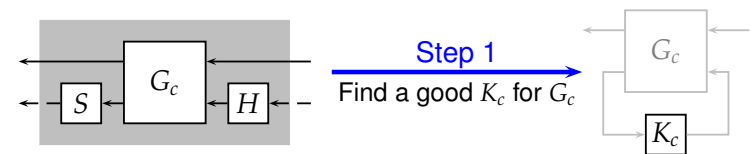
## Motivation of Study (1/3)

- We are interested in **design of  $K_d$**  for given  $G_c, S, H$
- How can we do it ?



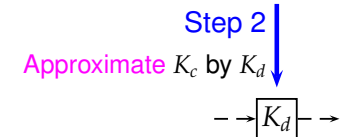
Sampled-Data Systems – p.7/22

## Motivation of Study (2/3): Conventional Design #1

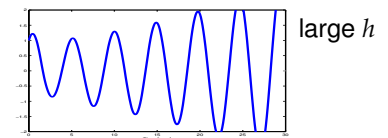
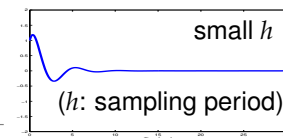


Tustin transformation is typical:

$$K_d[z] := K_c \left( \frac{2z-1}{h(z+1)} \right)$$

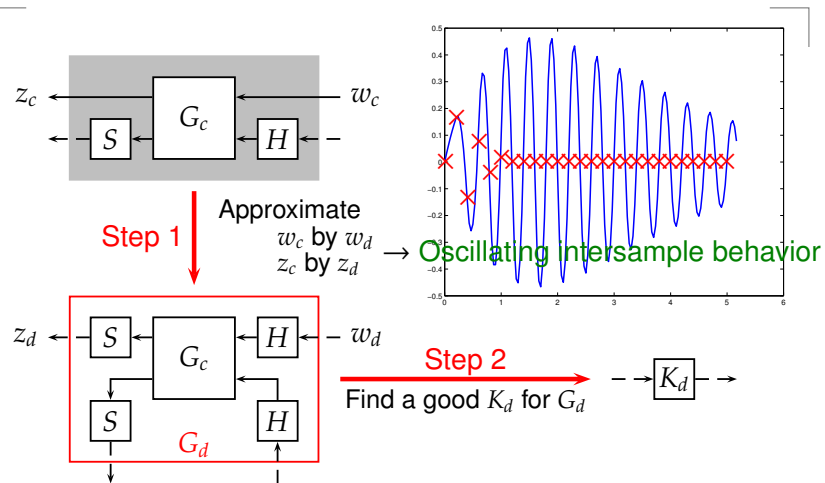


**Bad News:** Even stability is not guaranteed



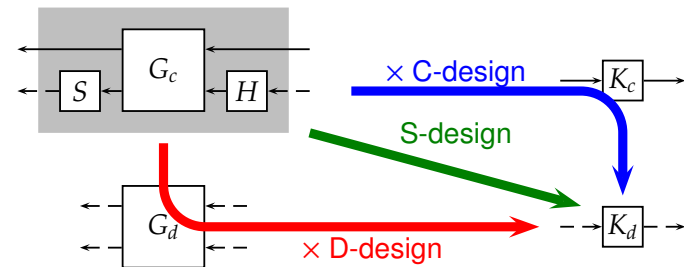
Sampled-Data Systems – p.8/22

## Motivation of Study (3/3): Conventional Design #2



Sampled-Data Systems – p.9/22

## Goal of This Course (1/2)

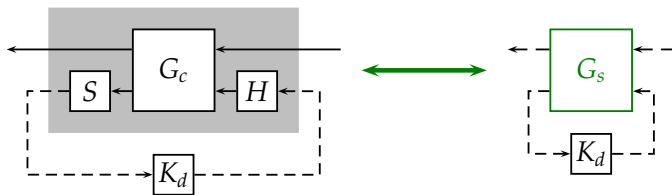


### Goal of This Course

- Design of  $K_d$  without approximation: Counterpart of LQ and  $H_\infty$  control
- Analysis as well

Sampled-Data Systems – p.10/22

## Goal of This Course (2/2)



We will derive a discrete-time system  $G_s$  which

- preserves **stability** and a specified **performance index**
- can depend on performance index
- is independent from  $K_d$
- as a result, decomposes design procedure into
  - Step 1: Find  $G_s$
  - Step 2: Find  $K_d$  for  $G_s$

Sampled-Data Systems – p.11/22

## Schedule

- L1 Sampled-Data Systems (1/6, 9.30–)  
Introduction
- L2 Review of Discrete-Time Control Theory (1/6, 11.00–)  
Discrete-Time LQ Optimal Control
- L3 Discrete-Time  $H_\infty$  Control (1/6, 13.30–)
- L4 Sampled-Data LQ Optimal Control (3/6, 9.30–)  
 $G_s$  for LQ Optimal Control
- L5 Time-Invariant Model of SDS (3/6, 11.00–)
- L6 Sampled-Data  $H_\infty$  Control (3/6, 13.00–)  
 $G_s$  for  $H_\infty$  Control
  - L2 + L4 → LQ Optimal Design
  - L3 + L6 →  $H_\infty$  Design

Sampled-Data Systems – p.12/22

## References

We assume

- experiences on state-space control theory
- basic knowledge on  $H_\infty$  control theory

for continuous-time systems

Supplemental readings for course material:

- Chen, Francis: Optimal Sampled-Data Control, Springer (1995)
- Zhou, Doyle, Glover: Robust and Optimal Control, Prentice Hall (1996)

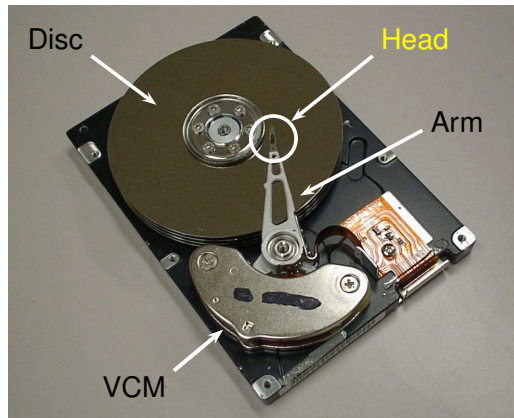
Sampled-Data Systems – p.13/22

## Outline

- Sampled-Data Systems: What and Why ?
- Design of Sampled-Data Systems
  - Conventional Methods
  - Goal of This Course
- Experimental Examples
  - Hard Disk Drive
  - Ultrasonic Motor

Sampled-Data Systems – p.14/22

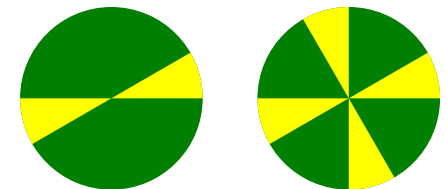
## Hard Disk Drive



Sampled-Data Systems – p.15/22

## Control of HDD

- High Capacity & High Performance
- Bad News:  $\text{Sampling Period} \propto \#$  (Position Information)



Position Information  
Data Area

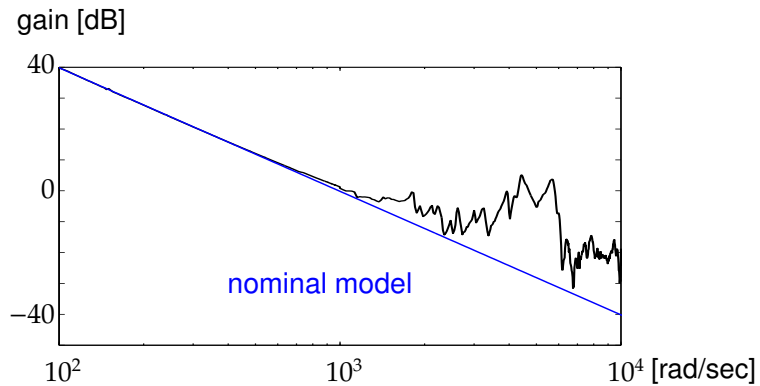
Less  
More

More  
Less

→ Larger Sampling Period is Better

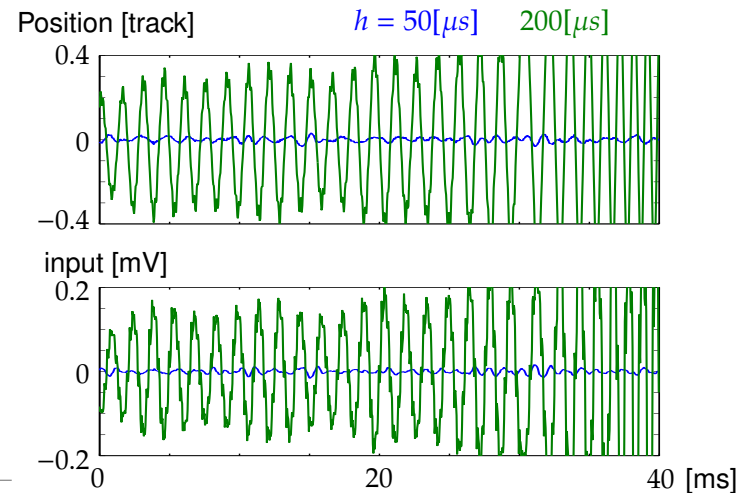
Sampled-Data Systems – p.16/22

## Frequency Response of HDD



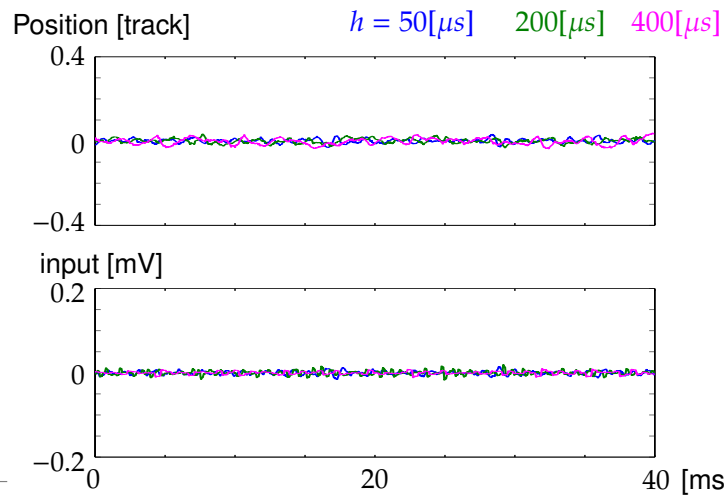
Sampled-Data Systems - p.17/22

## Following Control of HDD: C-design



Sampled-Data Systems - p.18/22

## Following Control of HDD: S-design



Sampled-Data Systems - p.19/22

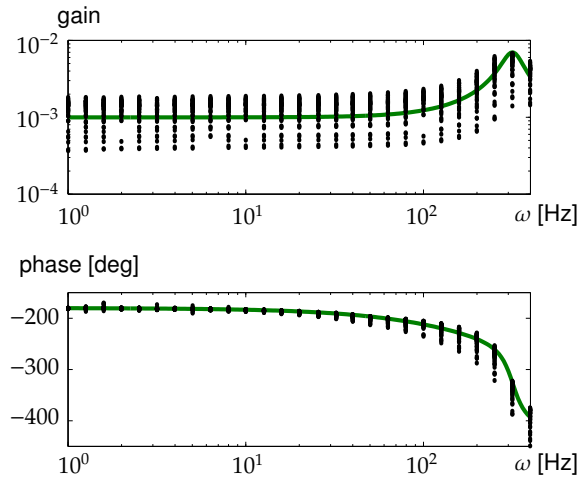
## Ultrasonic Motor



- Main application: AF SLR Camera
- Friction Nonlinearity

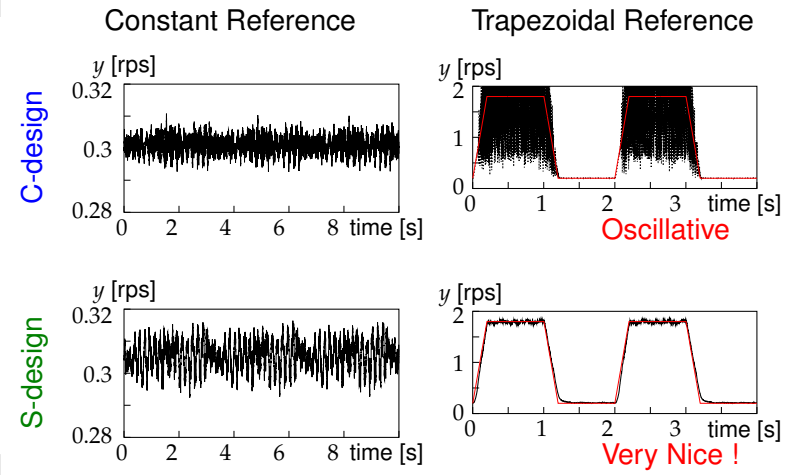
Sampled-Data Systems - p.20/22

## Frequency Response of USM



Sampled-Data Systems – p.21/22

## Speed Control of USM



Sampled-Data Systems – p.22/22