KAP. 1.7

1.64

a) $\lg \frac{7}{4} + \lg \frac{8}{7} = \lg\left(\frac{7}{4} \cdot \frac{8}{7}\right) = \lg 2$

c) $\lg 36 - 3\lg 6 = \lg 36 - \lg(6^3) = \lg\left(\frac{36}{6^3}\right) = \lg\left(\frac{6^2}{6^3}\right) = \lg \frac{1}{6}$

d) $\log_3 27 = \log_3(3^3) = 3 \cdot \log_3 3 = 3$

1.66 OBS! $\ln(a+b) \neq \ln a + \ln b$

TAG T.EX. $a = 1 = b$:

~~$\ln 2 \neq \ln(1+1) \neq \ln 1 + \ln 1 = 0 + 0 = 0$~~

V-L: $\ln(1+1) = \ln 2$

H-L: $\ln 1 + \ln 1 = 0 + 0 = 0$

EFTERSOM ATT $\ln 2 \neq 0$ SER VI ATT $\ln(1+1) \neq \ln 1 + \ln 1$.

1.70

a) $\lg x = \lg\left(e^{\ln x}\right) = \ln x \cdot \lg e$

b) $\log_2 x = \log_2\left(e^{\ln x}\right) = \ln x \cdot \log_2 e$

1.72 LÖS EKVATIONEN

a) $\ln x + \ln(x-1) = \ln 6$

$$\ln(x(x-1)) = \ln 6$$

$$x(x-1) = 6$$

$$x^2 - x = 6$$

$$(x-\tfrac{1}{2})^2 - \tfrac{1}{4} = 6$$

$$(x-\tfrac{1}{2})^2 = \frac{24+1}{4}$$

$$x = \tfrac{1}{2} \pm \sqrt{\tfrac{25}{4}} = \tfrac{1}{2} \pm \tfrac{5}{2} \implies x = \tfrac{6}{2} = 3 \text{ EL. } x = -\tfrac{4}{2} = -2$$

OBS! $\ln$ ÄR EJ DEFINIERAD FÖR $x \leq 0$ SÅ $x=-2$ ÄR EJ EN GILTIG ROT.

SVAR: $x = 3$

c) $2\ln(x-4) = \ln x + \ln 2$

$$\ln(x-4)^2 = \ln 2x$$

$$(x-4)^2 = 2x$$

LÅT $y := x-4$, DÅ $x = y+4$ OCH EKV. BLIR

$$y^2 = 2(y+4)$$

$$y^2 - 2y = 8$$

$$(y-1)^2 - 1 = 8$$

$$y = 1 \pm \sqrt{9} = 1 \pm 3 \implies y = 4 \text{ EL. } y = -2$$

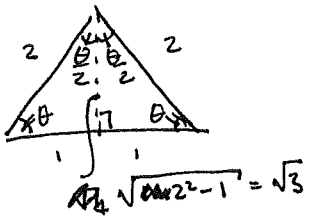
OM $y=4$ DÅ $x = 4+4 = 8$. OM $y=-2$ DÅ $x = -2+4 = 2$.

MEN OM $x=2$ SÅ $x-4 = 2-4 = -2$ SÅ $\ln(x-4)$ I EKV. ÄR EJ DEFINIERAD. D.V.S. $x=2$ ÄR EN FALSK ROT.

SVAR: $x = 8$

KAP. 1.9

1.94



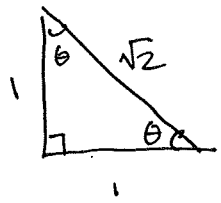
$3\theta = \pi \iff \theta = \frac{\pi}{3} \quad (60°)$

$$\cos\frac{\pi}{3} = \frac{1}{2}$$

$$\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$$

$$\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$$

$$\sin\frac{\pi}{6} = \frac{1}{2}$$



$$2\theta + \frac{\pi}{2} = \pi$$

$$2\theta = \frac{\pi}{2}$$

$$\theta = \frac{\pi}{4} \quad (45°)$$

$$\cos\frac{\pi}{2} = \frac{1}{\sqrt{2}} \left(= \frac{\sqrt{2}}{2}\right)$$

$$\sin\frac{\pi}{2} = \frac{1}{\sqrt{2}} \left(= \frac{\sqrt{2}}{2}\right)$$

1.95

$$\left.\begin{array}{l}\cos^2\alpha + \sin^2\alpha = 1 \\ \sin\alpha = 0.6\end{array}\right\} \Rightarrow \cos^2\alpha = 1 - 0.6^2 = 1 - 0.36 = 0.64$$

$$\cos\alpha = \pm\sqrt{0.64} = \pm 0.8$$

1.99 LÖS EKVATIONEN

a) $\cos x = \cos 3x$ (OBS: $\cos\alpha = \cos\beta \Leftrightarrow \begin{cases}\alpha = \beta + k\cdot 2\pi \\ \text{ELLER} \\ \alpha = -\beta + k\cdot 2\pi\end{cases}$

i) $x = 3x + k\cdot 2\pi$

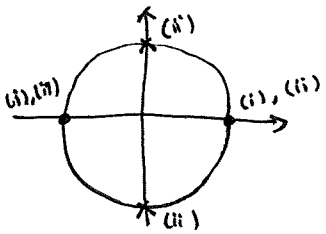
$-2x = k\cdot 2\pi$

$x = -k\cdot\pi$

ii) $x = -3x + k\cdot 2\pi$

$4x = k\cdot 2\pi$

$x = k\cdot\frac{\pi}{2}$



SVAR: $x = k\cdot\frac{\pi}{2}$, $k \in \mathbb{Z}$

1.100 LÖS EKVATIONEN

$\sin x = \cos 2x$

obs: $\sin x = \cos\left(x - \frac{\pi}{2}\right)$

$\cos\left(x - \frac{\pi}{2}\right) = \cos 2x$

i) $x - \frac{\pi}{2} = 2x + k \cdot 2\pi$

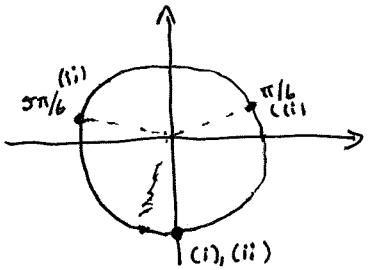
$-x = \frac{\pi}{2} + k \cdot 2\pi$

$x = -\frac{\pi}{2} - k \cdot 2\pi$

(ii) $x - \frac{\pi}{2} = -2x + k \cdot 2\pi$

$3x = \frac{\pi}{2} + k \cdot 2\pi$

$x = \frac{\pi}{6} + k \cdot \frac{2\pi}{3}$



SVAR: $x = \frac{\pi}{6} + k \cdot \frac{2\pi}{3}$, $k \in \mathbb{Z}$