$\boxed{1.103}$ BESTÄM AMPLITUD OCH FASFÖRSKJUTNING:

a) $f(x) = \cos x + \sin x = 1 \cdot \cos x + 1 \cdot \sin x$

$$= \sqrt{1^2+1^2} \cdot \left(\frac{1}{\sqrt{1^2+1^2}} \cos x + \frac{1}{\sqrt{1^2+1^2}} \cdot \sin x \right)$$

$$= \sqrt{2} \cdot \left(\frac{1}{\sqrt{2}} \cos x + \frac{1}{\sqrt{2}} \sin x \right)$$

$$= \sqrt{2} \cdot \left(\sin \frac{\pi}{4} \cdot \cos x + \cos \frac{\pi}{4} \cdot \sin x \right)$$

$$= \sqrt{2} \cdot \sin \left(x + \frac{\pi}{4} \right)$$

SVAR: AMPLITUD $\sqrt{2}$, FASFÖRSKJUTNING $\frac{\pi}{4}$.

b) $f(x) = \sqrt{3} \cdot \cos 2x - \sin 2x =$

$$= \sqrt{(\sqrt{3})^2 + (-1)^2} \cdot \left(\frac{\sqrt{3}}{2} \cos 2x - \frac{1}{2} \sin 2x \right)$$

$$= 2 \cdot \left(\sin \frac{\pi}{3} \cdot \cos 2x - \cos \frac{\pi}{3} \cdot \sin 2x \right)$$

$$= 2 \cdot \sin \left(\frac{\pi}{3} - 2x \right) = 2 \cdot \sin \left(-\left(2x - \frac{\pi}{3}\right) \right)$$

$$= 2 \cdot \sin \left(+\left(2x - \frac{\pi}{3}\right) + \pi \right)$$

$$= 2 \cdot \sin \left(2x + \frac{2\pi}{3} \right)$$

SVAR: AMPLITUD 2, FAS $\frac{2\pi}{3}$.

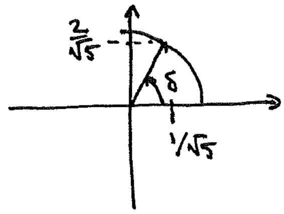
$\boxed{1.104}$ VISA ATT: $|\sin x + 2\cos x| \le \sqrt{5}$.

ANVÄND HJÄLPVINKELFORMELN:

$$\sin x + 2\cos x = \sqrt{1^2+2^2} \cdot \left(\frac{1}{\sqrt{5}} \sin x + \frac{2}{\sqrt{5}} \cos x \right)$$

$$= \sqrt{5} \cdot \left(\cos \delta \cdot \sin x + \sin \delta \cdot \cos x \right)$$

$$= \sqrt{5} \cdot \sin(\delta + x)$$



VI BEHÖVER INTE BESTÄMMA δ TY

NU FÅR VI ATT:

$$|\sin x + 2\cos x| = |\sqrt{5} \cdot \sin(x+\delta)| = \sqrt{5} \cdot |\sin(x+\delta)|$$

$$\le \sqrt{5} \cdot 1 \qquad (\text{OBS! } |\sin(x+\delta)| \le 1)$$

VILKET SKULLE VISAS.

$\boxed{1.106}$ LÖS EKVATIONEN:

a) $\sin x \cdot \cos x = \frac{1}{4}$

ADDITIONSFORMLER FÖR SINUS GER:

$$\sin(x+x) = \sin x \cdot \cos x + \cos x \cdot \sin x = 2\sin x \cdot \cos x$$

$$\Rightarrow \sin x \cdot \cos x = \frac{1}{2}\sin 2x$$

EKVATIONEN (a) KAN SKRIVAS OM:

$$\frac{1}{2}\cdot\sin 2x = \frac{1}{4}$$

$$\sin 2x = \frac{1}{2}$$

$$\Rightarrow \text{(i) } 2x = \frac{\pi}{6} + k\cdot 2\pi \qquad \text{EL. (ii) } 2x = \pi - \frac{\pi}{6} + k\cdot 2\pi$$

(i) $x = \frac{\pi}{12} + k\cdot\pi$ (ii) $x = \frac{5\pi}{12} + k\cdot\pi$

$x \in \left\{\frac{\pi}{12}, \frac{13\pi}{12}, \ldots\right\}$ $x \in \left\{\frac{5\pi}{12}, \frac{17\pi}{12}, \ldots\right\}$

INGET MÖNSTER SOM GER ATT MAN KAN SKRIVA BÄGGE PÅ EN FORMEL.

SVAR: $x = \frac{\pi}{12} + k\cdot\pi$ EL. $x = \frac{5\pi}{12} + k\cdot\pi$, $k \in \mathbb{Z}$

$\boxed{1.106}$

d) $\cos 2x + 3\cos x - 1 = 0$

ADDITIONSFORMEL + TRIGONOMETRISKA ETTAN GER:

$$\cos 2x = \cos(x+x) = \cos x \cdot \cos x - \sin x \cdot \sin x = \cos^2 x - \sin^2 x$$
$$= \cos^2 x - (1 - \cos^2 x) = 2\cos^2 x - 1$$

SÄTT IN I (d)

$$2\cos^2 x - 1 + 3\cos x - 1 = 0$$
$$2\cos^2 x + 3\cos x - 2 = 0$$

SUBSTITUERA, $y := \cos x$:

$$2y^2 + 3y - 2 = 0$$
$$y^2 + \frac{3}{2}y - 1 = 0$$
$$\left(y + \frac{3}{4}\right)^2 - \frac{9}{16} - 1 = 0$$
$$\left(y + \frac{3}{4}\right)^2 = \frac{25}{16}$$
$$y = -\frac{3}{4} \pm \sqrt{\frac{25}{16}} = -\frac{3}{4} \pm \frac{5}{4}, \quad \begin{cases} y_1 = -\frac{8}{4} = -2 \\ y_2 = \frac{2}{4} = \frac{1}{2} \end{cases}$$

$y = -2$:

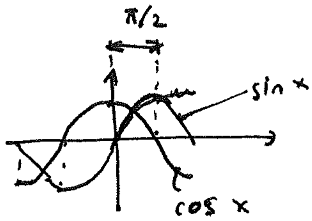
$\cos x = -2$ SAKNAR LÖSNING, TY $\cos x \geq -1$.

$y = \frac{1}{2}$:

$\cos x = \frac{1}{2} \implies x = \pm \frac{\pi}{3} + k \cdot 2\pi$

SVAR: $x = \pm \frac{\pi}{3} + k \cdot 2\pi, \quad k \in \mathbb{Z}$.

$\boxed{1.106}$



e) $\sin 4x = \cos 3x$

OBS: $\sin 4x = \cos\left(4x - \frac{\pi}{2}\right)$, VILKET GER

$$\cos\left(4x - \frac{\pi}{2}\right) = \cos 3x$$

$$\Rightarrow 4x - \frac{\pi}{2} = \pm 3x + k \cdot 2\pi$$

(i) $4x - \frac{\pi}{2} = 3x + k \cdot 2\pi$

$x = \frac{\pi}{2} + k \cdot 2\pi$

(ii) $4x - \frac{\pi}{2} = -3x + k \cdot 2\pi$

$7x = \frac{\pi}{2} + k \cdot 2\pi$

$x = \frac{\pi}{14} + k \cdot \frac{2\pi}{7}$

($\frac{\pi}{14} + k \cdot \frac{2\pi}{7} = \frac{\pi}{2}$ SAKNAR HELTALSLÖSNING SÅ VI KAN INTE SKRIVA (i) OCH (ii) PÅ EN FORMEL.)

SVAR: $x = \frac{\pi}{2} + k \cdot 2\pi$ EL. $x = \frac{\pi}{14} + k \cdot \frac{2\pi}{7}$, $k \in \mathbb{Z}$.