KAP. 1.10

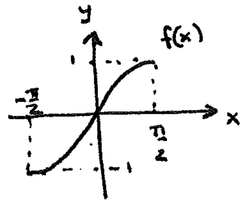
$\boxed{1.122}$

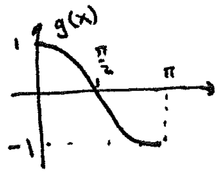
$f(x) := \sin x, \quad -\frac{\pi}{2} \le x \le \frac{\pi}{2}$

$g(x) := \cos x, \quad 0 \le x \le \pi$

$\arcsin y = f^{-1}(y), \quad -1 \le y \le 1$

$\arccos y = g^{-1}(y), \quad -1 \le y \le 1$





BESTÄM arcsin y OCH arccos y OM:

a) $y = 1$.

TÄNK PÅ arc-FUNKTIONERNA SOM INVERSER TILL f OCH g I FIGURERNA. DÅ SER MAN MED EN GÅNG ATT

$$\arcsin 1 = \frac{\pi}{2}$$

$$\arccos 1 = 0$$

c) $y = -\frac{\sqrt{3}}{2}$

OBS: $\sin\left(-\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2}$, $\quad \cos\left(\pi - \frac{\pi}{6}\right) = -\frac{\sqrt{3}}{2}$

VILKET GER

$$\arcsin\left(-\frac{\sqrt{3}}{2}\right) = -\frac{\pi}{3}$$

$$\arccos\left(-\frac{\sqrt{3}}{2}\right) = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$$

d) $y = \pi$

FRÅN FIGUR KAN UTLÄSAS ATT arcsin OCH arccos ENDAST ÄR DEFINIERADE FÖR $-1 \le y \le 1$.

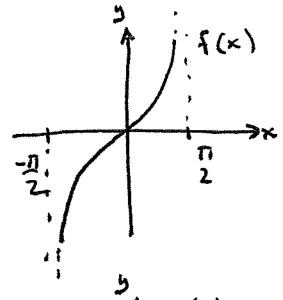
$\boxed{1.123}$

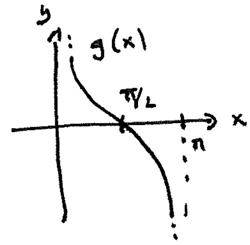
$f(x) := \tan x, \quad -\frac{\pi}{2} < x < \frac{\pi}{2}$

$g(x) := \cot x, \quad 0 < x < \pi$

$\arctan y = f^{-1}(y), \quad -\infty < y < \infty$

$\arccot y = g^{-1}(y), \quad -\infty < y < \infty$





BESTÄM arctan y OCH arccot y OM:

a) $y = 1$.

D.V.S. SÖK $x \in (-\frac{\pi}{2}, \frac{\pi}{2})$ S.A. $\tan x = 1$:

$$1 = \tan x = \frac{\sin x}{\cos x} \iff \cos x = \sin x \implies x = \frac{\pi}{4}$$

SÖK $x \in (0, \pi)$ S.A. $\cot x = 1$:

$$1 = \cot x = \frac{\cos x}{\sin x} \iff \sin x = \cos x \implies x = \frac{\pi}{4}$$

SVAR: $\arctan 1 = \frac{\pi}{4}$, $\operatorname{arccot} 1 = \frac{\pi}{4}$

b) $y = -1$

SÖK $x \in (-\frac{\pi}{2}, \frac{\pi}{2})$ S.A. $\tan x = -1$:

$$-1 = \tan x = \frac{\sin x}{\cos x} \iff -\cos x = \sin x \implies x = -\frac{\pi}{4}$$

SÖK $x \in (0, \pi)$ S.A. $\cot x = -1$:

$$-1 = \cot x = \frac{\cos x}{\sin x} \iff -\sin x = \cos x \implies x = \frac{3\pi}{4}$$

SVAR: $\arctan -1 = -\frac{\pi}{4}$, $\operatorname{arccot} -1 = \frac{3\pi}{4}$

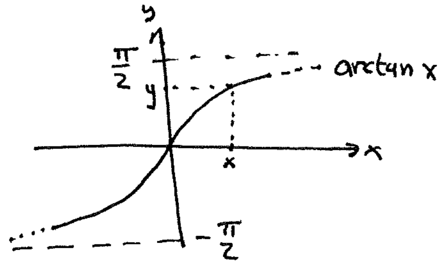
d) $y = 0$

FRÅN FIGUR SER VI DIREKT ATT $\arctan 0 = 0$

OCH $\operatorname{arccot} 0 = \frac{\pi}{2}$.

$\boxed{1.128}$ LÅT $y = \arctan x$, VISA ATT

$$\cos^2 y = \frac{1}{1+x^2}$$



$y = \arctan x \implies \tan y = x, \quad x \in \mathbb{R}$ (SE FIG. →)

SÅLEDES (ENL. DEF. AV tan, OCH ANV. TRIG. ETTAN):

$$x^2 = \tan^2 y = \frac{\sin^2 y}{\cos^2 y} = \frac{1-\cos^2 y}{\cos^2 y}$$

$$x^2 \cdot \cos^2 y = 1 - \cos^2 y$$

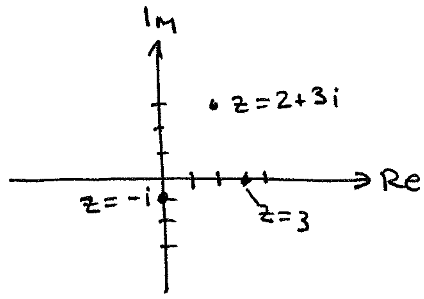
$$(x^2 + 1)\cos^2 y = 1$$

$$\cos^2 y = \frac{1}{1+x^2}$$

VILKET SKULLE VISAS.

APPENDIX A

A.1



a) $z = 2+3i$, $\text{Re}\, z = 2$, $\text{Im}\, z = 3$

c) $z = 3$, $\text{Re}\, z = 3$, $\text{Im}\, z = 0$

e) $z = -i$, $\text{Re}\, z = 0$, $\text{Im}\, z = -1$

A.2

a) $(1+i)+(-3-2i) = (1-3)+(1-2)i = -2-i$

g) $(1+i)(1-i) = 1^2 - i^2 = 1+1 = 2$

A.3

a) $\overline{1+i} = 1-i$

d) $(1+i)\overline{(1+i)} = (1+i)(1-i) = 1-i^2 = 1+1 = 2$ (OBS: $(1+i)\overline{(1+i)} = |1+i|^2$)

A.4

a) $\dfrac{1}{1+i} = \dfrac{\overline{1+i}}{(1+i)\overline{(1+i)}} = \dfrac{1-i}{|1+i|^2} = \dfrac{1-i}{2}$

A.5

a) OBS: $|z \cdot w| = |z| \cdot |w|$ SÅ $|z^{14}| = |z \cdot z^{13}| = |z| \cdot |z^{13}| = |z| \cdot |z \cdot z^{12}|$
$= |z| \cdot |z| \cdot |z^{12}| = \dots = |z|^{14}$

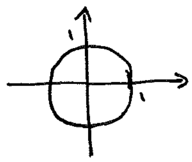
SÅ

$|(1-i)^{14}| = |1-i|^{14} = \left(\sqrt{1^2+(-1)^2}\right)^{14} = (\sqrt{2})^{14} = 2^7 = 128$

A.12

a) $|z| = 1 \iff \sqrt{x^2+y^2} = 1$ (CIRKELNS EKVATION)

$z = x+iy$

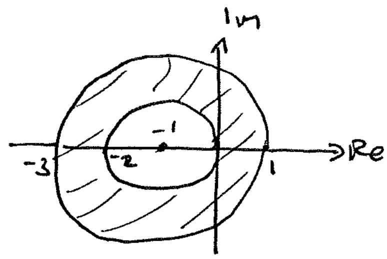


OBS: $|z| = |z-0|$

SÅ $|z|=1$ BESKRIVER ALLA PUNKTER PÅ AVSTÅND ETT FRÅN ORIGO — D.V.S. ENHETSCIRKELN.

g) $1 \leq |z+1| \leq 2$

D.V.S. AVSTÅNDET MELLAN z OCH -1 SKALL LIGGA MELLAN ETT OCH TVÅ



A.18

a) $|z| = \sqrt{2}$, $\arg z = \frac{\pi}{4}$

$\Rightarrow z = \sqrt{2} \cdot e^{i \cdot \frac{\pi}{4}} = \sqrt{2} \cdot \left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right) = \sqrt{2} \cdot \left(\frac{1}{\sqrt{2}} + i \cdot \frac{1}{\sqrt{2}}\right) = 1+i$

g) $|z| = 1$, $\arg z = -100\pi$

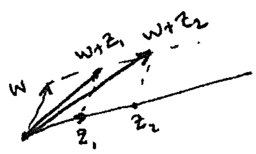
$\Rightarrow z = 1 \cdot e^{-i \cdot 100\pi} = 1 \cdot \left(\cos(-100\pi) + i\sin(-100\pi)\right) = 1 \cdot (1 + i \cdot 0) = 1$

A.22 $\arg z = \frac{\pi}{3}$, $\arg w = \frac{\pi}{4}$

a) $\arg(z \cdot w) = \arg z + \arg w = \frac{\pi}{3} + \frac{\pi}{4} = \frac{7\pi}{12}$

b) $\arg\left(\frac{z}{w}\right) = \arg z - \arg w = \frac{\pi}{3} - \frac{\pi}{4} = \frac{\pi}{12}$

c)



$\arg z_1 = \arg z_2$

MEN $\arg(w+z_1) \neq \arg(w+z_2)$!

SVAR: NEJ!