$\boxed{8.24}$ GIVET: YTAN $Y = \{(x,y,z) : z = x^2 + y^2, \; x^2+y^2 \le 1\}$

a) SKISSERA YTAN.

I CYLINDERKOORDINATER GES YTAN AV $z = \rho^2$, $\rho^2 \le 1$ SÅ YTAN GES AV EN ANDRAGRADSKURVA SOM ROTERAS RUNT Z-AXELN:

SKRIV PÅ PARAMETERFORM.
EXEMPELVIS SÅ KAN VI ANVÄNDA CYLINDER KOORD.:

$$\begin{cases} x = \rho \cdot \cos\varphi \\ y = \rho \cdot \sin\varphi \\ z = \rho^2 \end{cases} \qquad \begin{array}{l} 0 \le \rho \le 1 \\ 0 \le \varphi \le 2\pi \end{array}$$

MED $\bar{r}(\rho,\varphi) = (x,y,z)$ GES EN NORMAL AV $\bar{r}_\rho \times \bar{r}_\varphi$:

$$\bar{r}_\rho = (\cos\varphi, \sin\varphi, 2\rho), \qquad \bar{r}_\varphi = (-\rho\sin\varphi, \rho\cos\varphi, 0)$$

$$\bar{r}_\rho \times \bar{r}_\varphi = (-2\rho^2\cos\varphi, -2\rho^2\sin\varphi, \rho\cos^2\varphi + \rho\sin^2\varphi) =$$
$$= \rho(-2\rho\cos\varphi, -2\rho\sin\varphi, 1)$$

ALT. KAN VI HITTA NORMAL GENOM ATT SKRIVA Y PÅ IMPLICIT FORM OCH BERÄKNA GRADIENTEN:

$$Y = \{(x,y,z) : f(x,y,z) = 0, \; x^2+y^2 \le 1\}, \qquad f(x,y,z) = x^2+y^2-z$$

$$\nabla f(x,y,z) = (2x, 2y, -1) \quad \longleftarrow \text{ OCKSÅ EN NORMAL}$$

b) AREAN AV Y GES AV

$$\text{Area}(Y) = \iint |\bar{r}_\rho' \times \bar{r}_\varphi'| \, d\rho \, d\varphi$$

$$\left(|\bar{r}_\rho' \times \bar{r}_\varphi'| = \sqrt{4\rho^4\cos^2\varphi + 4\rho^4\sin^2\varphi + \rho^2} = \rho \cdot \sqrt{4\rho^2+1} \right)$$

$$= \int_{\rho=0}^{1} \left(\int_{\varphi=0}^{2\pi} \rho\sqrt{4\rho^2+1} \, d\varphi \right) d\rho = 2\pi \left[\frac{1}{12}(4\rho^2+1)^{3/2} \right]_0^1 = \frac{\pi}{6} \cdot \left((4 \cdot 1 + 1)^{3/2} - 1 \right)$$

$$= \underline{\frac{\pi}{6}\left(5^{3/2} - 1\right)}$$

c) BERÄKNA MASSAN OM MASSBELÄGGNINGEN GES AV $\sigma = x^2 = \rho^2\cos^2\varphi$:

$$m = \int_{\rho=0}^{1} \left(\int_{\varphi=0}^{2\pi} \rho^3\cos^2\varphi \cdot \sqrt{4\rho^2+1} \, d\varphi \right) d\rho = \int_{\varphi=0}^{2\pi} \frac{\cos 2\varphi + 1}{2} \int_{\rho=0}^{1} \rho^3\sqrt{4\rho^2+1} \, d\rho = \left\{ \begin{array}{l} t = 4\rho^2+1 \\ dt = 8\rho \, d\rho \end{array} \right\} =$$

$$= \frac{1}{2}\left[\frac{1}{2}\sin 2\varphi + \varphi \right]_0^{2\pi} \cdot \int_1^5 \frac{t-1}{4} \cdot \sqrt{t} \cdot \frac{1}{8} \, dt = \pi \cdot \frac{1}{32} \cdot \left[\frac{2}{5}t^{5/2} - \frac{2}{3}t^{3/2} \right]_1^5$$

$$= \frac{\pi}{16} \cdot \left(\frac{5^{5/2}}{5} - \frac{5^{3/2}}{3} - \frac{1}{5} + \frac{1}{3} \right) = \frac{\pi}{16} \cdot \frac{3 \cdot 5^2\sqrt{5} - 5 \cdot 5\sqrt{5} - 3 + 5}{3 \cdot 5} = \frac{\pi \cdot (2 \cdot 5^2\sqrt{5} + 2)}{16 \cdot 15} = \underline{\underline{\frac{\pi \cdot (25\sqrt{5} + 1)}{120}}}$$