POTENSLAGAR: $a > 0$, $b > 0$, α, β REELLA TAL

$$a^{\alpha} \cdot a^{\beta} = a^{\alpha+\beta} \qquad \left(a^{-\alpha} = \frac{1}{a^{\alpha}} \right)$$

$$(a^{\alpha})^{\beta} = a^{\alpha \cdot \beta} \qquad \left(a^{0} = 1 \right)$$

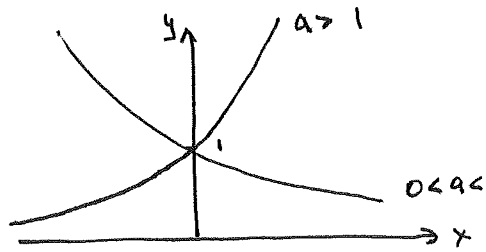
$$a^{\alpha} \cdot b^{\alpha} = (a \cdot b)^{\alpha}$$

! $(a+b)^{\alpha} \neq a^{\alpha} + b^{\alpha}$!

OBS! OM $a < 0$ SÅ ÄR a^{α} ENDAST DEFINIERAT FÖR HELTAL α!

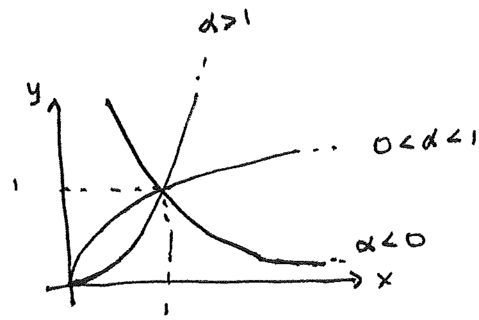
$$e^{\alpha} = \exp \alpha$$

EXPONENTIALFUNKTIONER



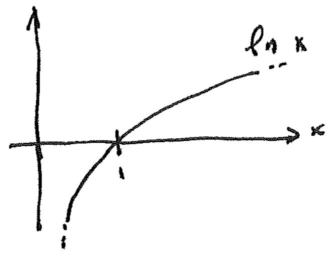
$y = a^{x}$

POTENSFUNKTIONER



$y = x^{\alpha}$

LOGARITM



LOGARITMLAGAR: $a, b > 0$

! $\log(a+b) \neq \log a + \log b$!

$$\log(a \cdot b) = \log a + \log b$$

$$\log\left(\frac{a}{b}\right) = \log a - \log b \qquad (\log 1 = 0)$$

$$\log a^{t} = t \cdot \log a \qquad (t \in \mathbb{R})$$

KAP. 1.6

1.53 FÖRENKLA

a) $$\frac{3^2\cdot 2^4}{6^3} = \frac{3^2\cdot 2^4}{(2\cdot 3)^3} = \frac{3^2\cdot 2^4}{2^3\cdot 3^3} = 3^{2-3}\cdot 2^{4-3} = 3^{-1}\cdot 2^1 = \underline{\underline{\frac{2}{3}}}$$

c) $$\left(\sqrt{64}\right)^{2/3} = \left(64^{1/2}\right)^{2/3} = 64^{\frac{1}{2}\cdot\frac{2}{3}} = 64^{1/3} = \left(2^6\right)^{1/3} = 2^{6/3} = 2^2 = \underline{\underline{4}}$$

e) $$2^{(2^3)} = 2^8 = \underline{\underline{256}}$$

1.54 FÖRENKLA

b) $$\frac{a\sqrt{a}}{\sqrt[3]{a^2}} = \frac{a\cdot a^{1/2}}{(a^2)^{1/3}} = \frac{a^{1+1/2}}{a^{2/3}} = \frac{a^{3/2}}{a^{2/3}} = a^{\frac{3}{2}-\frac{2}{3}} = a^{\frac{9-4}{6}} = \underline{\underline{a^{5/6}}}$$ (om $a>0$)

d) $$\frac{\sqrt[4]{a^3\sqrt{a}}}{\sqrt[8]{1/a}} = \frac{(a^3\cdot a^{1/2})^{1/4}}{(a^{-1})^{1/8}} = \frac{\left(a^{3+\frac{1}{2}}\right)^{1/4}}{a^{-1/8}} = a^{1/8}\cdot\left(a^{7/2}\right)^{1/4} = a^{1/8}\cdot a^{7/8} = a^{\frac{1}{8}+\frac{7}{8}} = a^1 = \underline{\underline{a}}$$ (om $a>0$)

f) $$\left(ab\sqrt[4]{a^3/\sqrt{b\sqrt{b}}}\right)^2 = \left(ab\cdot\left[\frac{a^3}{(b\cdot b^{1/2})^{1/2}}\right]^{1/4}\right)^2 = \left\{(ab)^2\cdot\left[\frac{a^3}{(b^{3/2})^{1/2}}\right]^{1/2}\right.$$

$$= (ab)^2\cdot\frac{a^{3/2}}{b^{\frac{3}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}}} = a^2\cdot b^2\cdot a^{3/2}\cdot b^{-3/8} = a^{2+\frac{3}{2}}\cdot b^{2-\frac{3}{8}}$$

$$= \underline{\underline{a^{7/2}\cdot b^{13/8}}}$$

1.61 FÖRENKLA:

a) $3^x + 3^{x+1} = 3^x + 3^x \cdot 3 = 3^x \cdot (1+3) = \underline{\underline{4 \cdot 3^x}}$

d) $\frac{1}{e^x} + \frac{1}{e^{x+1}} = \frac{1}{e^x} + \frac{1}{e^x \cdot e} = \frac{1}{e^x} \cdot \left(1 + \frac{1}{e}\right)$

$= \frac{1}{e^x} \cdot \frac{e+1}{e} = \frac{1}{e^{x+1}} \cdot (e+1) \quad \left(= (1+e) \cdot e^{-(x+1)}\right)$

1.63 LÖS EKVATIONEN:

a) $2^x \cdot 3^{x-2} = 4$

$2^x \cdot 3^x \cdot 3^{-2} = 4$

$(2 \cdot 3)^x = 4 \cdot 3^2$

$6^x = 36$

$x = 2$

SVAR: $x=2$

b) $4^x - 6 \cdot 2^x + 8 = 0$

$2^x \cdot 2^x - 6 \cdot 2^x + 8 = 0$ LÅT $y := 2^x$

$y^2 - 6y + 8 = 0$

$(y-3)^2 - 3^2 + 8 = 0$

$(y-3)^2 = 1$

$y = 3 \pm 1$

OM $y = 2$ DÅ $x = 1$ $(2^1 = 2)$

OM $y = 4$ DÅ $x = 2$ $(2^2 = 4)$

SVAR: $x_1 = 1$, $x_2 = 2$

KAP. 1.7

$\boxed{1.64}$

a) $\lg \frac{7}{4} + \lg \frac{8}{7} = \lg\left(\frac{7}{4} \cdot \frac{8}{7}\right) = \lg 2$

c) $\lg 36 - 3\lg 6 = \lg 36 - \lg(6^3) = \lg\left(\frac{36}{6^3}\right) = \lg\left(\frac{6^2}{6^3}\right) = \lg \frac{1}{6}$

d) $\log_3 27 = \log_3(3^3) = 3 \cdot \log_3 3 = 3$

$\boxed{1.66}$ OBS! $\ln(a+b) \neq \ln a + \ln b$

TAG T.EX. $a = 1 = b$:

V-L: $\ln(1+1) = \ln 2$

H-L: $\ln 1 + \ln 1 = 0 + 0 = 0$

EFTERSOM ATT $\ln 2 \neq 0$ SER VI ATT $\ln(1+1) \neq \ln 1 + \ln 1$.

$\boxed{1.70}$

a) $\lg x = \lg\left(e^{\ln x}\right) = \ln x \cdot \lg e$

b) $\log_2 x = \log_2\left(e^{\ln x}\right) = \ln x \cdot \log_2 e$

1.72 LÖS EKVATIONEN

a) $\ln x + \ln(x-1) = \ln 6$

$$\ln(x(x-1)) = \ln 6$$

$$x(x-1) = 6$$

$$x^2 - x = 6$$

$$(x-\tfrac{1}{2})^2 - \tfrac{1}{4} = 6$$

$$(x-\tfrac{1}{2})^2 = \frac{24+1}{4}$$

$$x = \tfrac{1}{2} \pm \sqrt{\tfrac{25}{4}} = \tfrac{1}{2} \pm \tfrac{5}{2} \Rightarrow x = \tfrac{6}{2} = 3$$

EL.

$$x = -\tfrac{4}{2} = -2$$

OBS! ln ÄR EJ DEFINIERAD FÖR $x \leq 0$ SÅ $x=-2$ ÄR EJ EN GILTIG ROT.

SVAR: $x = 3$

c) $2\ln(x-4) = \ln x + \ln 2$

$$\ln(x-4)^2 = \ln 2x$$

$$(x-4)^2 = 2x$$

LÅT $y := x-4$, DÅ $x = y+4$ OCH EKV. BLIR

$$y^2 = 2(y+4)$$

$$y^2 - 2y = 8$$

$$(y-1)^2 - 1 = 8$$

$$y = 1 \pm \sqrt{9} = 1 \pm 3 \Rightarrow y = 4 \text{ EL. } y = -2$$

OM $y=4$ DÅ $x = 4+4 = 8$. OM $y=-2$ DÅ $x = -2+4 = 2$.

MEN OM $x=2$ SÅ $x-4 = 2-4 = -2$ SÅ $\ln(x-4)$ I EKV. ÄR EJ DEFINIERAD. D.V.S. $x=2$ ÄR EN FALSK ROT.

SVAR: $x = 8$

$\boxed{1.73}$ RADIOAKTIVT SÖNDERFALL UPPFYLLER

$$m(t) = m(0) \cdot e^{-\lambda t}$$

$m(t)$: MASSA VID TID t

λ : SÖNDERFALLSKONSTANT

LÅT T BETECKNA HALVERINGSTID.

BESTÄM SAMBAND MELLAN T OCH λ.

VID TID $t = T$ HAR MASSAN HALVERATS, D.V.S.

$$m(T) = \frac{m(0)}{2}$$

$$m(0) \cdot e^{-\lambda \cdot T} = \frac{m(0)}{2}$$

$$e^{-\lambda \cdot T} = \frac{1}{2}$$

$$2 = e^{\lambda \cdot T}$$

$$\ln 2 = \lambda \cdot T \cdot \ln e$$

$$T = \frac{\ln 2}{\lambda} \quad \text{EL.} \quad \lambda = \frac{\ln 2}{T}$$

SVAR: $\lambda = \dfrac{\ln 2}{T}$