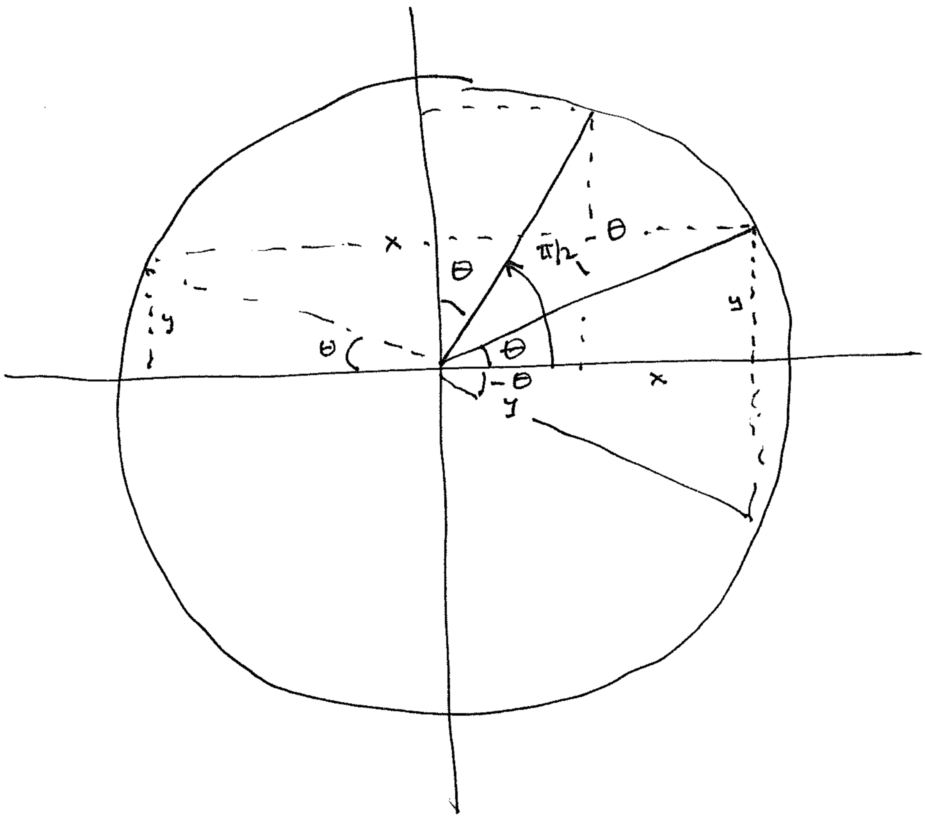
ENHETSCIRKELN : $x^2+y^2=1$



$x = \cos\theta$

$y = \sin\theta$

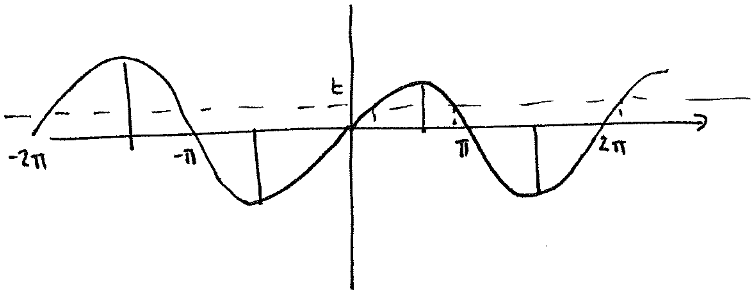
$x = \sin\left(\frac{\pi}{2}-\theta\right)$

$y = \cos\left(\frac{\pi}{2}-\theta\right)$

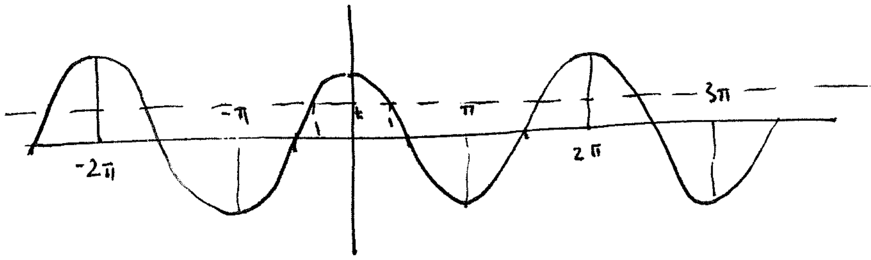
$x^2+y^2=1 \Rightarrow \cos^2\theta + \sin^2\theta = 1$

$\cos(-\theta) = x = \cos\theta$

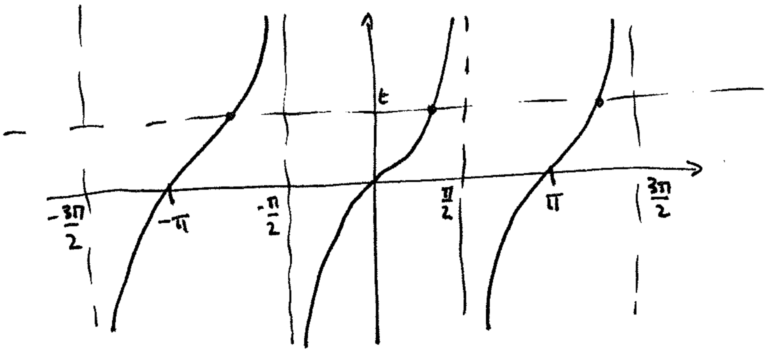
$\sin(\pi-\theta) = y = \sin\theta$



$$t = \sin\alpha = \sin\beta \quad \Longrightarrow \quad \begin{cases} \alpha = \beta & + 2\pi k \\ \alpha = \pi - \beta & + 2\pi k \end{cases}$$



$$t = \cos\alpha = \cos\beta \quad \Longrightarrow \quad \begin{cases} \alpha = \beta & + 2\pi k \\ \alpha = -\beta & + 2\pi k \end{cases}$$



$$t = \tan\alpha = \tan\beta \quad \Longrightarrow \quad \alpha = \beta + k \cdot \pi$$

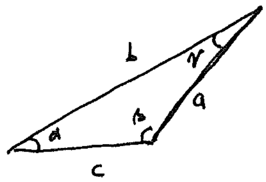
ADDITIONSSATSER:

(i) $\cos(x+y) = \cos x \cdot \cos y - \sin x \cdot \sin y$

(ii) $\sin(x+y) = \sin x \cdot \cos y + \cos x \cdot \sin y$

TRIGONOMETRISKA ETTAN: (FÖLJER FRÅN (i) MED $y=-x$)

$$(\cos x)^2 + (\sin x)^2 = 1$$



COSINUSSATSEN:

$$a^2 = b^2 + c^2 - 2bc \cdot \cos\alpha$$

SINUSSATSEN:

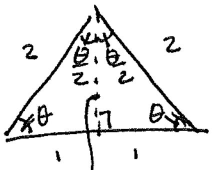
$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c}$$

AREASATSEN:

$$\text{AREA} = \frac{1}{2} b \cdot c \cdot \sin\alpha$$

KAP. 1.9

$\boxed{1.94}$

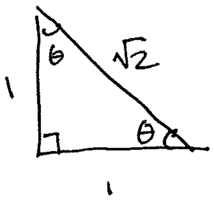


$\sqrt{2^2-1} = \sqrt{3}$

$3\theta = \pi \iff \theta = \frac{\pi}{3}$ (60°)

$$\boxed{\begin{aligned} \cos\frac{\pi}{3} &= \frac{1}{2} \\ \cos\frac{\pi}{6} &= \frac{\sqrt{3}}{2} \end{aligned}}$$

$$\boxed{\begin{aligned} \sin\frac{\pi}{3} &= \frac{\sqrt{3}}{2} \\ \sin\frac{\pi}{6} &= \frac{1}{2} \end{aligned}}$$



$$2\theta + \frac{\pi}{2} = \pi$$

$$2\theta = \frac{\pi}{2}$$

$$\theta = \frac{\pi}{4} \qquad (45^\circ)$$

$$\boxed{\begin{aligned} \cos\frac{\pi}{2} &= \frac{1}{\sqrt{2}} \quad \left(= \frac{\sqrt{2}}{2}\right) \\ \sin\frac{\pi}{2} &= \frac{1}{\sqrt{2}} \quad \left(= \frac{\sqrt{2}}{2}\right) \end{aligned}}$$

1.95

$$\left.\begin{array}{l}\cos^2\alpha+\sin^2\alpha=1\\ \sin\alpha=0.6\end{array}\right\}\Rightarrow \begin{array}{l}\cos^2\alpha = 1-0.6^2 = 1-0.36 = 0.64\\ \cos\alpha = \pm\sqrt{0.64} = \pm 0.8\end{array}$$

1.99 LÖS EKVATIONEN

a) $\cos x = \cos 3x$ (OBS: $\cos\alpha = \cos\beta \Leftrightarrow \begin{cases}\alpha = \beta + k\cdot 2\pi \\ \text{ELLER} \\ \alpha = -\beta + k\cdot 2\pi\end{cases}$

i) $x = 3x + k\cdot 2\pi$

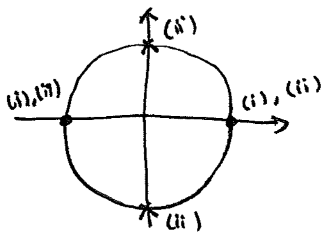
$-2x = k\cdot 2\pi$

$x = -k\cdot\pi$

ii) $x = -3x + k\cdot 2\pi$

$4x = k\cdot 2\pi$

$x = k\cdot\frac{\pi}{2}$



SVAR: $x = k\cdot\frac{\pi}{2}$, $k\in\mathbb{Z}$

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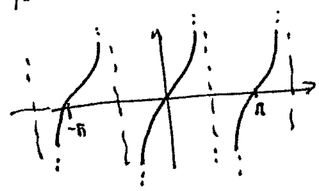
b) $\tan x = \tan 3x$ (OBS: $\tan\alpha = \tan\beta \Leftrightarrow \alpha = \beta + k\cdot\pi$)



$x = 3x + k\cdot\pi$

$x = -\frac{k\cdot\pi}{2}$

D.V.S. $x = 0, \pm\frac{\pi}{2}, \pm\pi, \pm\frac{3\pi}{2}, \pm 2\pi, \dots$

MEN KOM IHÅG ATT $\tan x$ EJ DEF. FÖR $x = \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \dots$

SÅ DESSA RÖTTER ÄR FALSKA. DE ENDA GILTIGA RÖTTERNA ÄR ALLTSÅ $x = 0, \pm\pi, \pm 2\pi, \dots$

SVAR: $x = k\cdot\pi$, $k\in\mathbb{Z}$.

$\boxed{1.100}$ LÖS EKVATIONEN

$\sin x = \cos 2x$ OBS: $\sin x = \cos\left(x - \frac{\pi}{2}\right)$

$\cos\left(x-\frac{\pi}{2}\right) = \cos 2x$

i) $x - \frac{\pi}{2} = 2x + k\cdot 2\pi$

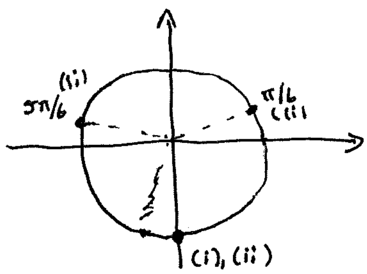
$-x = \frac{\pi}{2} + k\cdot 2\pi$

$x = -\frac{\pi}{2} - k\cdot 2\pi$

(ii) $x - \frac{\pi}{2} = -2x + k\cdot 2\pi$

$3x = \frac{\pi}{2} + k\cdot 2\pi$

$x = \frac{\pi}{6} + k\cdot \frac{2\pi}{3}$

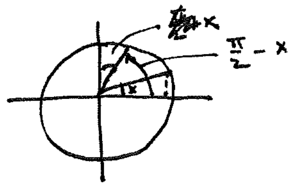


SVAR: $x = \frac{\pi}{6} + k\cdot\frac{2\pi}{3}$, $k \in \mathbb{Z}$

$\boxed{1.101}$ LÖS EKVATIONEN $\cos 4x = \sin x$

SKRIV OM EKV. SÅ ATT VI BARA HAR COS, ELLER BARA SIN.

$$\sin x = \cos\left(\frac{\pi}{2} - x\right)$$



DETTA GER:

$$\cos 4x = \cos\left(\frac{\pi}{2} - x\right)$$

$$\Rightarrow \begin{cases} 4x = \frac{\pi}{2} - x + 2\pi k & \text{(i)} \\ 4x = -\left(\frac{\pi}{2} - x\right) + 2\pi k & \text{(ii)} \end{cases} \quad (k \in \mathbb{Z})$$

i) $\quad 5x = \frac{\pi}{2} + 2\pi k \quad \Rightarrow \quad x = \frac{\pi}{10} + \frac{2\pi k}{5}$

ii) $\quad 4x = -\frac{\pi}{2} + x + 2\pi k \quad \Rightarrow \quad 3x = -\frac{\pi}{2} + 2\pi k \quad \Rightarrow \quad x = -\frac{\pi}{6} + \frac{2\pi k}{3}$

SVAR: $\quad x = \frac{\pi}{10} + \frac{2\pi k}{5}$ EL. $x = -\frac{\pi}{6} + \frac{2\pi k}{3}$ (DÄR $k \in \mathbb{Z}$).

ALTERNATIV LÖSNING:

ANVÄND $\quad \cos 4x = \sin\left(\frac{\pi}{2} - 4x\right)$

$$\sin\left(\frac{\pi}{2} - 4x\right) = \sin x$$

$$\Rightarrow \begin{cases} \frac{\pi}{2} - 4x = x + 2\pi k & \text{(i)} \\ \frac{\pi}{2} - 4x = \pi - x + 2\pi k & \text{(ii)} \end{cases} \quad (k \in \mathbb{Z})$$

i) $\quad \frac{\pi}{2} - 4x = x + 2\pi k \quad \Rightarrow \quad -5x = -\frac{\pi}{2} + 2\pi k \quad \Rightarrow \quad x = \frac{\pi}{10} - \frac{2\pi k}{5}$

ii) $\quad \frac{\pi}{2} - 4x = \pi - x + 2\pi k \quad \Rightarrow \quad -3x = -\frac{\pi}{2} + \pi + 2\pi k \quad \Rightarrow \quad x = -\frac{\pi}{6} - \frac{2\pi k}{3}$

DETTA GER SAMMA LÖSNINGAR SOM OVAN (TY $k = 0, \pm 1, \pm 2, \dots$ SÅ TECKNET FRAMFÖR $2\pi k/5$ OCH $2\pi k/3$ HAR INGEN BETYDELSE).

$\boxed{1.103}$ BESTÄM AMPLITUD OCH FASFÖRSKJUTNING:

a) $f(x) = \cos x + \sin x = 1 \cdot \cos x + 1 \cdot \sin x$

$= \sqrt{1^2+1^2} \cdot \left(\frac{1}{\sqrt{1^2+1^2}} \cos x + \frac{1}{\sqrt{1^2+1^2}} \cdot \sin x \right)$

$= \sqrt{2} \cdot \left(\frac{1}{\sqrt{2}} \cos x + \frac{1}{\sqrt{2}} \sin x \right)$

$= \sqrt{2} \cdot \left(\sin \frac{\pi}{4} \cdot \cos x + \cos \frac{\pi}{4} \cdot \sin x \right)$

$= \sqrt{2} \cdot \sin \left(x + \frac{\pi}{4} \right)$

SVAR: AMPLITUD $\sqrt{2}$, FASFÖRSKJUTNING $\frac{\pi}{4}$.

b) $f(x) = \sqrt{3} \cdot \cos 2x - \sin 2x =$

$= \sqrt{(\sqrt{3})^2 + (-1)^2} \cdot \left(\frac{\sqrt{3}}{2} \cos 2x - \frac{1}{2} \sin 2x \right)$

$= 2 \cdot \left(\sin \frac{\pi}{3} \cdot \cos 2x - \cos \frac{\pi}{3} \cdot \sin 2x \right)$

$= 2 \cdot \sin \left(\frac{\pi}{3} - 2x \right) = 2 \cdot \sin \left(-\left(2x - \frac{\pi}{3}\right) \right)$

$= 2 \cdot \sin \left(+\left(2x - \frac{\pi}{3}\right) + \pi \right)$

$= 2 \cdot \sin \left(2x + \frac{2\pi}{3} \right)$

SVAR: AMPLITUD 2, FAS $\frac{2\pi}{3}$.

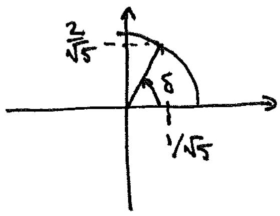
$\boxed{1.104}$ VISA ATT: $|\sin x + 2\cos x| \le \sqrt{5}$.

ANVÄND HJÄLPVINKELFORMELN:

$$\sin x + 2\cos x = \sqrt{1^2+2^2} \cdot \left(\frac{1}{\sqrt{5}} \sin x + \frac{2}{\sqrt{5}} \cos x \right)$$

$$= \sqrt{5} \cdot (\cos \delta \cdot \sin x + \sin \delta \cdot \cos x)$$

$$= \sqrt{5} \cdot \sin(\delta + x)$$



VI BEHÖVER INTE BESTÄMMA δ TY

NU FÅR VI ATT:

$$|\sin x + 2\cos x| = |\sqrt{5} \cdot \sin(x+\delta)| = \sqrt{5} \cdot |\sin(x+\delta)|$$

$$\le \sqrt{5} \cdot 1 \qquad (\text{OBS! } |\sin(x+\delta)| \le 1)$$

VILKET SKULLE VISAS.

$\boxed{1.106}$

d) $\cos 2x + 3\cos x - 1 = 0$

ADDITIONSFORMEL + TRIGONOMETRISKA ETTAN GER:

$$\cos 2x = \cos(x+x) = \cos x \cdot \cos x - \sin x \cdot \sin x = \cos^2 x - \sin^2 x$$
$$= \cos^2 x - (1-\cos^2 x) = 2\cos^2 x - 1$$

SÄTT IN I (d)

$$2\cos^2 x - 1 + 3\cos x - 1 = 0$$
$$2\cos^2 x + 3\cos x - 2 = 0$$

SUBSTITUERA, $y := \cos x$:

$$2y^2 + 3y - 2 = 0$$
$$y^2 + \frac{3}{2}y - 1 = 0$$
$$\left(y + \frac{3}{4}\right)^2 - \frac{9}{16} - 1 = 0$$
$$\left(y + \frac{3}{4}\right)^2 = \frac{25}{16}$$
$$y = -\frac{3}{4} \pm \sqrt{\frac{25}{16}} = -\frac{3}{4} \pm \frac{5}{4}, \quad \begin{cases} y_1 = -\frac{8}{4} = -2 \\ y_2 = \frac{2}{4} = \frac{1}{2} \end{cases}$$

$y = -2$:

$\cos x = -2$ SAKNAR LÖSNING, TY $\cos x \geq -1$.

$y = \frac{1}{2}$:

$\cos x = \frac{1}{2} \implies x = \pm \frac{\pi}{3} + k \cdot 2\pi$

SVAR: $x = \pm \frac{\pi}{3} + k \cdot 2\pi, \quad k \in \mathbb{Z}$.

$\boxed{1.106}$ LÖS EKVATIONEN:

a) $\sin x \cdot \cos x = \frac{1}{4}$

ADDITIONSFORMLER FÖR SINUS GER:

$$\sin(x+x) = \sin x \cdot \cos x + \cos x \cdot \sin x = 2 \sin x \cdot \cos x$$

$$\Rightarrow \quad \sin x \cdot \cos x = \frac{1}{2} \sin 2x$$

EKVATIONEN (a) KAN SKRIVAS OM:

$$\frac{1}{2} \cdot \sin 2x = \frac{1}{4}$$

$$\sin 2x = \frac{1}{2}$$

$$\Rightarrow \text{(i) } 2x = \frac{\pi}{6} + k \cdot 2\pi \qquad \text{EL. (ii) } 2x = \pi - \frac{\pi}{6} + k \cdot 2\pi$$

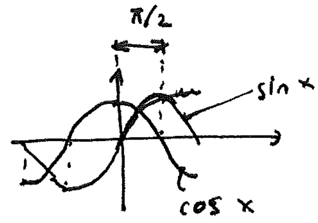
$$\text{(i)} \quad x = \frac{\pi}{12} + k \cdot \pi \qquad \text{(ii)} \quad x = \frac{5\pi}{12} + k \cdot \cancel{2}\pi$$

$$x \in \left\{ \frac{\pi}{12}, \frac{13\pi}{12}, \ldots \right\} \qquad x \in \left\{ \frac{5\pi}{12}, \frac{17\pi}{12}, \ldots \right\}$$

INGET MÖNSTER SOM
GER ATT MAN KAN SKRIVA
BÄGGE PÅ EN FORMEL.

SVAR: $x = \frac{\pi}{12} + k \cdot \pi$ EL. $x = \frac{5\pi}{12} + k \cdot \pi$, $k \in \mathbb{Z}$

$\boxed{1.106}$



e) $\sin 4x = \cos 3x$

OBS: $\sin 4x = \cos\left(4x - \frac{\pi}{2}\right)$, VILKET GER

$$\cos\left(4x - \frac{\pi}{2}\right) = \cos 3x$$

$$\Rightarrow 4x - \frac{\pi}{2} = \pm 3x + k \cdot 2\pi$$

(i) $4x - \frac{\pi}{2} = 3x + k \cdot 2\pi$

$x = \frac{\pi}{2} + k \cdot 2\pi$

(ii) $4x - \frac{\pi}{2} = -3x + k \cdot 2\pi$

$7x = \frac{\pi}{2} + k \cdot 2\pi$

$x = \frac{\pi}{14} + k \cdot \frac{2\pi}{7}$

($\frac{\pi}{14} + k \cdot \frac{2\pi}{7} = \frac{\pi}{2}$ SAKNAR HELTALSLÖSNING SÅ VI KAN INTE SKRIVA (i) OCH (ii) PÅ EN FORMEL.)

SVAR: $x = \frac{\pi}{2} + k \cdot 2\pi$ EL. $x = \frac{\pi}{14} + k \cdot \frac{2\pi}{7}$, $k \in \mathbb{Z}$.

$\boxed{1.112}$ VISA ATT $\dfrac{1-\sin\theta}{1+\sin\theta} = \tan^2\left(\frac{\pi}{4}-\frac{\theta}{2}\right)$

VI RÄKNAR "BAKLÄNGES":

$$\tan^2\left(\frac{\pi}{4}-\frac{\theta}{2}\right) = \left[\frac{\sin\left(\frac{\pi}{4}-\frac{\theta}{2}\right)}{\cos\left(\frac{\pi}{4}-\frac{\theta}{2}\right)}\right]^2$$

$$= \left[\frac{\cancel{\sin\frac{\pi}{4}}\cdot\cos\frac{\theta}{2} - \cancel{\cos\frac{\pi}{4}}\cdot\sin\frac{\theta}{2}}{\cancel{\cos\frac{\pi}{4}}\cdot\cos\frac{\theta}{2} + \cancel{\sin\frac{\pi}{4}}\cdot\sin\frac{\theta}{2}}\right]^2$$

(ADDITIONSSATS FÖR sin OCH cos)

$$= \left[\frac{\cos\frac{\theta}{2} - \sin\frac{\theta}{2}}{\cos\frac{\theta}{2} + \sin\frac{\theta}{2}}\right]^2$$

$\left(\sin\frac{\pi}{4} = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}\right)$

$$= \frac{\cos^2\frac{\theta}{2} - 2\cos\frac{\theta}{2}\cdot\sin\frac{\theta}{2} + \sin^2\frac{\theta}{2}}{\cos^2\frac{\theta}{2} + 2\cos\frac{\theta}{2}\cdot\sin\frac{\theta}{2} + \sin^2\frac{\theta}{2}}$$

$$= \frac{1 - 2\cdot\cos\frac{\theta}{2}\cdot\sin\frac{\theta}{2}}{1 + 2\cdot\cos\frac{\theta}{2}\cdot\sin\frac{\theta}{2}}$$

(TRIG. ETTAN)

$$= \frac{1 - \sin\left(\frac{\theta}{2}+\frac{\theta}{2}\right)}{1 + \sin\left(\frac{\theta}{2}+\frac{\theta}{2}\right)}$$

(ADDITIONSSATS FÖR sin)

$$= \frac{1-\sin\theta}{1+\sin\theta}$$

VILKET SKULLE VISAS. □