M35, 13 - 15

6.4: 1, 3, 7, 9

6.5: 1, 3, 5, 12, 15, 16

6.6: 2, 3, 5, 6, 7, 12, 17, 18, 21, 22

7.4: 1, 4, 5, 6, 8, 9

7.5: 3, 5, 16, 28, 29, 31

$\bar{x}' = \bar{f}(t, \bar{x})$

$\bar{x}' = A\bar{x}$

---

① ALLA LÖSN.

FUND. LÖSN. M.

$\{y_i\}_{i=1}^{n}$

LIND. OBER.

$y = c_1 y_1 + \dots + c_n y_n$

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② HITTA LÖSN.

LÖSN. $A\bar{v} = \lambda \bar{v}$

$\bar{v}' = \lambda \bar{v}$

$y' = \lambda y$ $y = e^{\lambda t}$

(3)

$$\begin{cases} \dfrac{dx_1}{dt} = 2x_1 - 3x_2 \\ \dfrac{dx_2}{dt} = -3x_1 + 2x_2 \end{cases} \qquad \bar{x}' = \begin{pmatrix} 2 & -3 \\ -3 & 2 \end{pmatrix} \bar{x}$$

↑ SYMMETRISK ⇒ DIAGONALISERBAR!

SÖK EGENVÄRDEN:

$$0 = \begin{vmatrix} 2-\lambda & -3 \\ -3 & 2-\lambda \end{vmatrix} = (2-\lambda)^2 - 9 = (2-\lambda+3)(2-\lambda-3) = (-\lambda+5)(-\lambda-1)$$

$$\lambda_1 = 5 \qquad \lambda_2 = -1$$

SÖK EGENVEKTORER:

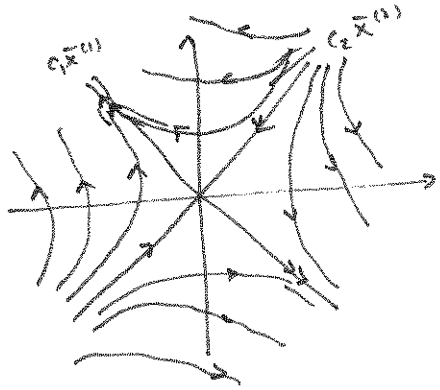
$$\begin{pmatrix} -3 & -3 \\ -3 & -3 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0 \quad \Rightarrow \quad x_1 = -x_2 \qquad \bar{v}^{(1)} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$\begin{pmatrix} 3 & -3 \\ -3 & 3 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0 \quad \Rightarrow \quad x_1 = x_2 \qquad \bar{v}^{(2)} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

ALLMÄN LÖSNING:

$$\bar{x} = c_1 \cdot e^{5t}\begin{pmatrix} 1 \\ -1 \end{pmatrix} + c_2 \cdot e^{-t} \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix} = c_1 \bar{x}^{(1)} + c_2 \bar{x}^{(2)}$$

a) OM $t \to \infty$ SÅ $e^{-t} \to 0$. D.V.S. $\bar{x}(t)$ NÄRMAR SIG $c_1 \bar{x}^{(1)}$.
OM $t \to -\infty$ SÅ $e^{5t} \to 0$. D.V.S. $\bar{x}(t)$ NÄRMAR SIG $c_2 \bar{x}^{(2)}$.



b) SÖK $\bar{x}(t)$ S.A. $\bar{x}(0) = \begin{pmatrix} 5 \\ -1 \end{pmatrix}$

$$\begin{pmatrix} 5 \\ -1 \end{pmatrix} = c_1\begin{pmatrix} 1 \\ -1 \end{pmatrix} + c_2\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\begin{cases} c_1 + c_2 = 5 \\ -c_1 + c_2 = -1 \end{cases} \qquad \begin{aligned} 2c_2 &= 4 \\ c_2 &= 2 \\ c_1 &= 5 - c_2 = 3 \end{aligned}$$

SVAR: $\bar{x} = 2\bar{x}^{(1)} + 3\bar{x}^{(2)}$

c) FÖR VILKA B.V. GÄLLER ATT $\bar{x}(t) \to 0$?

FRÅN FASDIAGRAMMET SER MAN ATT ENDAST ~~[illegible]~~ PUNKTER PÅ DEN INSTABILA MÅNGFALDEN $s \cdot \bar{x}^{(2)}$, $s \in \mathbb{R}$ GÅR MOT NOLL.

(4) LÖS INTEGRALEKVATIONEN

$$e^{-t} = y(t) + \int_0^t (t-u)y(u)\,du$$

OBS ATT HL INNEHÅLLER EN FALTNING:

$$e^{-t} = y(t) + g * y(t) \quad , \quad g(t) = t$$

LAPLACETRANSFORMEN GER

$$\frac{1}{s+1} = \mathcal{L}\{e^{-t}\} = \mathcal{L}\{y(t) + g * y(t)\}$$

$$= \mathcal{L}\{y(t)\} + \mathcal{L}\{g * y\} \quad \text{(LINEARITET)}$$

$$= Y(s) + \mathcal{L}\{g\}\mathcal{L}\{y(t)\} \quad \text{(EGENSKAP AV } *\text{)}$$

$$= Y(s) + \frac{1}{s^2} \cdot Y(s)$$

BRYT UT $Y(s)$:

$$Y(s) = \frac{1}{s+1} \cdot \left(1 + \frac{1}{s^2}\right)^{-1} = \frac{s^2}{(s+1)(s^2+1)}$$

$$= \frac{A}{s+1} + \frac{B}{s^2+1} + \frac{Cs}{s^2+1} = \frac{A(s^2+1) + B(s+1) + Cs(s+1)}{(s^2+1)(s+1)}$$

$$\Rightarrow \begin{cases} A + C = 1 \\ A + B = 0 \\ B + C = 0 \end{cases} \Rightarrow \begin{array}{l} (A+C) + (A+B) - (B+C) = 1 + 0 - 0 \\ 2A = 1 \end{array}$$

$$\Rightarrow \begin{cases} A = \frac{1}{2} \\ B = -\frac{1}{2} \\ C = \frac{1}{2} \end{cases}$$

$$Y(s) = \frac{1}{2} \cdot \left(\frac{1}{s+1} - \frac{1}{s^2+1} + \frac{s}{s^2+1} \right)$$

$$\Rightarrow \quad y(t) = \frac{1}{2} \cdot \left(\mathcal{L}^{-1}\left\{\frac{1}{s+1}\right\} - \mathcal{L}^{-1}\left\{\frac{1}{s^2+1}\right\} + \mathcal{L}^{-1}\left\{\frac{s}{s^2+1}\right\} \right) \quad \text{(LINEARITET)}$$

$$= \frac{1}{2} \cdot \left(e^{-t} - \sin t + \cos t \right)$$

SVAR: $y(t) = \dfrac{e^{-t} - \sin t + \cos t}{2}$

KONTROLLERA!