$a > 0 \qquad f_a(t) = f(a \cdot t)$

$$\Rightarrow \quad \hat{f}_a(\omega) = \int_{-\infty}^{\infty} f(a\cdot t)\cdot e^{-i\omega t}\,dt = \begin{Bmatrix} s = a\cdot t \\ ds = a\cdot dt \end{Bmatrix} = \int_{-\infty}^{\infty} f(s)\cdot e^{-i\frac{\omega}{a}\cdot t}\cdot \frac{ds}{a}$$

$$= \frac{1}{a}\cdot \hat{f}\left(\frac{\omega}{a}\right) \qquad \boxed{\hat{f}_a(\omega) = \frac{1}{a}\cdot \hat{f}\left(\frac{\omega}{a}\right)}$$

$\boxed{7.19}$ HITTA EN LÖSNING TILL

$$\int_{-\infty}^{\infty} f(t-y)\cdot e^{-|y|}\,dy = \frac{4}{3}e^{-|t|} - \frac{2}{3}e^{-2|t|}$$

LÅT $g(t) = e^{-|t|}$. DÅ SER VI ATT VL ÄR EN FALTNING, SÅ EKV. KAN SKRIVAS

$$f*g(t) = \frac{4}{3}\cdot g(t) - \frac{2}{3}\cdot g(2t)$$

FOURIERTRANSFORMERA!

$$\left.\begin{aligned} \widehat{f*g}(\omega) &\overset{\text{SATS 7.6}}{=} \hat{f}(\omega)\cdot\hat{g}(\omega) \\ &\overset{\text{UNDRE STEGET}}{=} \frac{4}{3}\hat{g}(\omega) - \frac{2}{3}\cdot\frac{1}{2}\cdot\hat{g}\left(\frac{\omega}{2}\right) \end{aligned}\right\} \Rightarrow \hat{f}(\omega) = \frac{4}{3} - \frac{1}{3}\cdot\frac{\hat{g}(\omega/2)}{\hat{g}(\omega)}$$

$\hat{g}(\omega) = \dfrac{2}{1+\omega^2}$ (TABELL)

$$\hat{f}(\omega) = \frac{4}{3} - \frac{1}{3}\cdot\frac{1+\omega^2}{1+\omega^2/4} = \frac{4+\omega^2-1-\omega^2}{3\cdot(1+\omega^2/4)} = \frac{1}{1+\omega^2/4} = \hat{g}_2(\omega)$$

$$\Rightarrow \quad f(t) = g_2(t) = g(2t) = e^{-2|t|}$$

<u>**SVAR**</u>: $f(t) = e^{-2|t|}$

PLANCHEREL:

$$\boxed{\begin{array}{l}\int f(t)\overline{g(t)}\,dt = \frac{1}{2\pi}\int \hat{f}(t)\overline{\hat{g}(t)}\,dt \\ \int f(t)\hat{g}(t)\,dt = \frac{1}{2\pi}\int \hat{f}(\omega)g(\omega)\,d\omega\end{array}}$$

$$\int_{-\infty}^{\infty} |f(t)|^2\,dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |\hat{f}(\omega)|^2\,d\omega$$

$\boxed{7.41}$

$$f(t) = \begin{cases} \sin t, & |t| < \pi \\ 0, & \text{ANNARS} \end{cases}$$

BESTÄM FOURIERTRANSFORMEN FÖR f OCH BERÄKNA

$$\int_{-\infty}^{\infty} \frac{\sin^2 \pi t}{(t^2-1)^2}\,dt.$$

F-TRANSF.:

$$\hat{f}(\omega) = \int_{-\infty}^{\infty} f(t)\,e^{-i\omega t}\,dt = \int_{-\pi}^{\pi} \sin t \cdot e^{-i\omega t}\,dt$$

$$= \left[\sin t \cdot \frac{e^{-i\omega t}}{-i\omega}\right]_{-\pi}^{\pi} - \int_{-\pi}^{\pi} \cos t \cdot \frac{e^{-i\omega t}}{-i\omega}\,dt$$

$$= \frac{1}{i\omega}\cdot\left(\left[\cos t \cdot \frac{e^{-i\omega t}}{-i\omega}\right]_{-\pi}^{\pi} + \int_{-\pi}^{\pi} \sin t \cdot \frac{e^{-i\omega t}}{-i\omega}\,dt\right)$$

$$= \frac{1}{i\omega}\cdot\left(\frac{-e^{-i\omega\pi} + e^{i\omega\pi}}{-i\omega} - \frac{1}{i\omega}\cdot\hat{f}(\omega)\right)$$

$$= \frac{1}{\omega^2}\cdot\left(2i\cdot\sin\omega\pi + \hat{f}(\omega)\right)$$

$$\Rightarrow \hat{f}(\omega)\cdot\left(1 - \frac{1}{\omega^2}\right) = \frac{2i\sin\omega\pi}{\omega^2} \Rightarrow \boxed{\hat{f}(\omega) = \frac{2i\cdot\sin\omega\pi}{\omega^2-1}}$$

PLANCHEREL:

$$\int_{-\infty}^{\infty} \left|\frac{\sin\omega\pi}{\omega^2-1}\right|^2 d\omega = \int_{-\infty}^{\infty} \left|\frac{\hat{f}(\omega)}{2}\right|^2 d\omega = \frac{1}{4}\int_{-\infty}^{\infty} |\hat{f}(\omega)|^2\,d\omega =$$

$$= \frac{1}{4}\cdot 2\pi\cdot\int_{-\infty}^{\infty} |f(t)|^2\,dt = \frac{\pi}{2}\int_{-\pi}^{\pi} \sin^2 t\,dt$$

$$= \pi\cdot\int_0^{\pi} \frac{1-\cos 2t}{2}\,dt = \frac{\pi}{2}\cdot\left(\pi - \left[\frac{\sin 2t}{2}\right]_0^{\pi}\right) = \frac{\pi^2}{2}$$

<u>SVAR:</u> $\hat{f}(\omega) = \dfrac{2i\cdot\sin\omega\pi}{\omega^2-1}$, $\displaystyle\int_{-\infty}^{\infty} \frac{\sin^2\pi t}{(t^2-1)^2}\,dt = \frac{\pi^2}{2}$

3.44

a) $a_n = 2^{-n}$

$$Z\{a_n\} = \sum_{n=0}^{\infty} 2^{-n} z^{-n} = \sum_{n=0}^{\infty} (2z)^{-n} = \frac{1}{1-\frac{1}{2z}} = \frac{2z}{2z-1}$$

b) $a_n = n \cdot 3^n$

$$Z\{a_n\} = \sum_{n=0}^{\infty} \underbrace{\frac{n \cdot 3^n}{z^n}} = \sum_{n=0}^{\infty} \frac{d}{dz}\left\{\left(\frac{z}{3}\right)^{-n}\right\} \cdot (-z)$$

$$\left(n \cdot (z/3)^{-n} = \frac{d}{dz} (z/3)^{-n} \cdot \frac{z}{3} \cdot 3 \cdot (-1) \right)$$

$$= -z \cdot \frac{d}{dz} \left\{ \sum_{n=0}^{\infty} \left(\frac{z}{3}\right)^{-n} \right\} = -z \cdot \frac{d}{dz} \left\{ \frac{1}{1-\frac{3}{z}} \right\}$$

$$= -z \cdot (-1) \cdot \left(1-\frac{3}{z}\right)^{-2} \cdot (+3 \cdot z^{-2}) = \frac{3z}{z^2 \cdot \left(1-\frac{3}{z}\right)^2} = \frac{3z}{(z-3)^2}$$

c) $a_n = n^2 \, 2^n$

$$A(z) = \sum_{n \geq 0} \frac{n^2 \cdot 2^n}{z^n} = \sum_{n \geq 0} (n(n+1) - n) \cdot (z/2)^{-n}$$

$$= \sum_{n \geq 0} z^2 \cdot \frac{d^2}{dz^2}\left\{\left(\frac{z}{2}\right)^{-n}\right\} + z \cdot \frac{d}{dz}\left\{\left(\frac{z}{2}\right)^{-n}\right\}$$

$\boxed{3.44}$

c) $a_n = n^2 \cdot 2^n$

OBS!

$$\frac{d}{dz}\left\{\left(\frac{a}{z}\right)^n\right\} = n\left(\frac{a}{z}\right)^{n-1} \cdot (-a \cdot z^{-2}) = -n \cdot z^{-1} \cdot \left(\frac{a}{z}\right)^n$$

$$\frac{d^2}{dz^2}\left\{\left(\frac{a}{z}\right)^n\right\} = a^n \cdot \frac{d}{dz}\left\{-n \cdot z^{-n-1}\right\} = -a^n \cdot n \cdot \left(-(n+1)z^{-n-2}\right) = n(n+1)z^{-2}\left(\frac{a}{z}\right)^n$$

$$n \cdot \left(\frac{a}{z}\right)^n = -z \cdot \frac{d}{dz}\left\{\left(\frac{a}{z}\right)^n\right\}$$

$$n(n+1) \cdot \left(\frac{a}{z}\right)^n = z^2 \cdot \frac{d}{dz}\left\{\left(\frac{a}{z}\right)^n\right\}$$

DETTA GER

$$A(z) = \sum n^2 \cdot \left(\frac{2}{z}\right)^n = \sum n(n+1)\left(\frac{2}{z}\right)^n - n \cdot \left(\frac{2}{z}\right)^n$$

$$= z^2 \cdot \frac{d^2}{dz^2}\left\{\sum\left(\frac{2}{z}\right)^n\right\} + z \cdot \frac{d}{dz}\left\{\sum\left(\frac{2}{z}\right)^n\right\}$$

$$= z^2 \cdot \frac{d^2}{dz^2}\left\{\frac{1}{1-\frac{2}{z}}\right\} + z \cdot \frac{d}{dz}\left\{\frac{1}{1-\frac{2}{z}}\right\}$$

$$= z^2 \cdot \frac{d}{dz}\left\{-\left(1-\frac{2}{z}\right)^{-2} \cdot 2z^{-2}\right\} + z \cdot \left(-\left(1-\frac{2}{z}\right)^{-2} \cdot 2z^{-2}\right)$$

$$= -\frac{2z}{(z-2)^2} - 2z^2 \cdot \frac{(z-2)^2 - z \cdot 2(z-2)}{(z-2)^4}$$

$$= -\frac{2z}{(z-2)^2} - 2z^2 \cdot \frac{z-2-2z}{(z-2)^3} = -\frac{2z}{(z-2)^2} + \frac{2z^2(z+2)}{(z-2)^3}$$

2014-01-14

2

$a_0 = a_1 = 0$

$a_{n+2} - 5a_{n+1} + 6a_n = 2 \quad , \qquad n = 0, 1, 2, \ldots$

BESTÄM EN FORMEL FÖR a_n, $n = 0, 1, 2, \ldots$

$$\sum_{n\geq 0} a_{n+2} z^{-n} - 5\sum_{n\geq 0} a_{n+1} z^{-n} + 6 \sum_{n \geq 0} a_n z^{-n} = 2 \sum_{n\geq 0} z^{-n}$$

$$z^2 \sum_{n\geq 0} a_{n+2} z^{-(n+2)} - 5z \sum_{n\geq 0} a_{n+1} z^{-(n+1)} + 6 \cdot A(z) = 2 \cdot \frac{1}{1-\frac{1}{z}}$$

$$z^2 \cdot \left(A(z) - a_0 - a_1 z^{-1} \right) - 5z \cdot \left(A(z) - a_0 \right) + 6A(z) = \frac{2z}{z-1}$$

$$\left(z^2 - 5z + 6 \right) A(z) = \frac{2z}{z-1} \qquad \left[(z-2)(z-3) = z^2 - 5z + 6 \right.$$

$$A(z) = \frac{2z}{(z-1)(z-2)(z-3)}$$

$$\frac{A(z)}{2z} = \frac{c_0}{z-1} + \frac{c_1}{z-2} + \frac{c_2}{z-3} = \frac{c_0(z-2)(z-3) + c_1(z-1)(z-3) + c_2(z-1)(z-2)}{(z-1)(z-2)(z-3)}$$

$$\begin{cases} z^2 \cdot (c_0 + c_1 + c_2) = 0 \\ z \cdot (-5c_0 - 4c_1 - 3c_2) = 0 \\ 6c_0 + 3c_1 + 2c_2 = 1 \end{cases} \quad c_0 - c_1 - c_2 = 1 \qquad 2c_0 = 1$$

$$c_1 + c_2 = -\frac{1}{2}$$

$$4c_1 + 3c_2 = -\frac{5}{2} \qquad c_1 = -\frac{5}{2} + \frac{3}{2} = -1$$

$$\begin{cases} c_0 = \frac{1}{2} \\ c_1 = -1 \\ c_2 = \frac{1}{2} \end{cases}$$

$$\frac{A(z)}{z} = \frac{1}{z-1} - \frac{2}{z-2} + \frac{1}{z-3}$$

$$A(z) = \frac{1}{1-\frac{1}{z}} - \frac{2}{1-\frac{2}{z}} + \frac{1}{1-\frac{3}{z}} = \sum_{n\geq 0} z^{-n} - 2 \cdot \sum_{n\geq 0} (z/2)^{-n} + \sum_{n \geq 0} (z/3)^{-n}$$

$$= \sum_{n\geq 0} \underbrace{(1 - 2\cdot 2^n + 3^n)}_{a_n} z^{-n}$$

SVAR: $a_n = 1 - 2^{n+1} + 3^n, \quad n = 0, 1, 2, \ldots$