LEKTION 13 29/2/2008 SF1646

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$\boxed{9.3}$ $I = \int_\gamma (x^2+xy)\,dx + (y^2-xy)\,dy$

a) $\begin{cases} x = 2t \\ y = 2t \end{cases}$ $t \in [0,1]$

$I = \int_0^1 \{ (4t^2+4t^2)\cdot 2 + (4t^2-4t^2)\cdot 2 \}\,dt = \int_0^1 16t^2\,dt = \frac{16}{3}$

b) $\begin{cases} x = 2t \\ y = 2t^2 \end{cases}$ $t \in [0,1]$

$I = \int_0^1 \{ (4t^2+4t^3)\cdot 2 + (4t^4-4t^3)\cdot 4t \}\,dt = \int_0^1 (16t^5 - 16t^4 + 8t^3 + 8t^2)\,dt$

$= 8\cdot\left[2\cdot\frac{1}{6} - 2\cdot\frac{1}{5} + \frac{1}{4} + \frac{1}{3}\right) = 2\cdot\frac{20-24+15+20}{15} = \frac{62}{15}$

c) $\begin{cases} x = 2t \\ y = 0 \end{cases}$ $t \in [0,1]$

$I_1 = \int_0^1 (4t^2)\cdot 2\,dt = 8\cdot\frac{1}{3}$

$\begin{cases} x = 2 \\ y = 2t \end{cases}$ $t \in [0,1]$

$I_2 = \int_0^1 \{(4+4t)\,0 + (4t^2-4t)2\}\,dt$

$= 8\cdot\left(\frac{1}{3}-\frac{1}{2}\right) = -\frac{8}{6} = -\frac{4}{3}$

$I = I_1 + I_2 = \frac{4}{3}$

$\boxed{9.4}$ $\begin{cases} x = t \\ y = t^2 \end{cases}$ $t \in [1,2]$

$\int_\gamma y \ln\frac{x^2}{y}\,dx - \frac{x}{y}\,dy = \int_1^2 \left(t^2 \cdot \ln 1 \cdot 1 - \frac{1}{t}\cdot 2t\right)dt = -2$

$\boxed{9.5}$ $\bar{F}(x,y) = (1,-2)$. γ LINJE FRÅN $(0,1)$ TILL $(2,2)$

W = ARBETE = $\int_\gamma \bar{F}\cdot d\bar{r}$, $\bar{r} = (2t, 1+t)$, $t \in [0,1]$; $\bar{r}' = (2,1)$

$W = \int_0^1 (1,-2)\cdot(2,1)\,dt = 0$

$\boxed{9.6}$ grad $f(x,y)$ ÄR VINKELRÄT MOT NIVÅKURVAN $f(x,y)=c$ i (x,y),

SÅLEDES ÄR INTEGRALEN NOLL

SAMMANFATTNING – LEKTION 13

BEGREPP: – KURVINTEGRALER, DEFINITION 1 (s.328)