

Let  $L(H)$  denote the algebra of bounded operators on a separable Hilbert space  $H$ . By an *asymptotic representation* of a  $C^*$ -algebra  $A$  on  $H$  we mean an asymptotic homomorphism  $\mu = (\mu_t)_{t \in [0, \infty)} : A \rightarrow L(H)$ . By using asymptotic representations instead of genuine ones we introduce two new tensor norms on the algebraic tensor product  $A \odot D$  of two  $C^*$ -algebras  $A$  and  $D$ , which respect asymptotic homomorphisms.

Let  $H_1, H_2$  be separable Hilbert spaces and let  $\mu = (\mu_t)_{t \in [0, \infty)} : A \rightarrow L(H_1), \nu = (\nu_t)_{t \in [0, \infty)} : D \rightarrow L(H_2)$  be two equicontinuous asymptotic representations. For a finite sum  $c = \sum_i a_i \odot d_i \in A \odot D$ ,  $a_i \in A$ ,  $d_i \in D$ , put  $\|c\|_{\mu, \nu} = \limsup_{t \rightarrow \infty} \|\sum_i \mu_t(a_i) \otimes \nu_t(d_i)\|$ . Define the *asymptotic tensor norm* by  $\|c\|_\sigma = \sup_{\mu, \nu} \|c\|_{\mu, \nu}$ , where we take the supremum over all pairs  $(\mu, \nu)$  of asymptotic representations of  $A$  and  $D$  respectively. We define also the *left asymptotic tensor norm*  $\|\cdot\|_\lambda$  on  $A \odot D$  by taking the supremum over all pairs  $(\mu, \nu)$ , where  $\mu$  is an asymptotic representation of  $A$  and  $\nu$  is a genuine representation of  $D$ .

Denote by  $A \otimes_\lambda D$  and  $A \otimes_\sigma D$  the  $C^*$ -algebras obtained by completing  $A \odot D$  with respect to the norm  $\|\cdot\|_\lambda$  and  $\|\cdot\|_\sigma$  respectively.

If  $\phi = (\phi_t)_{t \in [0, \infty)} : A_1 \rightarrow A_2$  and  $\psi = (\psi_t)_{t \in [0, \infty)} : D_1 \rightarrow D_2$  are asymptotic homomorphisms then their tensor product  $\phi_t \otimes \psi_t$  extends to an asymptotic homomorphism from  $A_1 \otimes_\sigma D_1$  to  $A_2 \otimes_\sigma D_2$ .

**Theorem.** The tensor norm  $\|\cdot\|_\lambda$  differs both from the minimal and the maximal tensor norms. The tensor norm  $\|\cdot\|_\sigma$  differs from the minimal tensor norm.