Compact perturbations of linear differential equations in locally convex spaces

S. A. Shkarin

By $\mathcal{P}$ we denote the class of locally convex spaces E for which for any continuous linear operator $T : E \to E$ solvability (in $[0,1]$) of the Cauchy problem $\dot{x} = Tx$, $x(0) = x_0$ for any $x_0 \in E$ implies solvability of the problem $\dot{x}(t) = Tx(t) + f(t, x(t))$, $x(0) = x_0$ for any $x_0 \in E$ and any continuous map $f : [0,1] \times E \to E$ with relatively compact image.

Herzog and Lemmert [1] have proven that class $\mathcal{P}$ contains all Fréchet space. We prove the following stronger theorem.

Theorem 1. *Class $\mathcal{P}$ contains duals of separable metrizable locally convex spaces with the pre-compact convergence topology, sequentially complete compactly regular countable inductive limits of Fréchet spaces and sequentially complete compactly regular spaces, being countable inductive limits of countable projective limits of countable inductive limits of separable Fréchet spaces.*

From Theorem 1 it follows that class $\mathcal{P}$ contains the spaces $D(R^n)$ of infinitely differentiable functions with compact support, $S'(R^n)$ of Schwarz distributions, $D'(R^n)$ of generalized functions and $A(R^n)$ of real-analytic functions.

1. G. Herzog and R. Lemmert, *Nonlinear fundamental systems for linear differential equations in Fréchet spaces,* Demonstratio Math., 2000, **33**, 313–318