SPECTRAL ANALYSIS OF A NON-SELFADJOINT DIFFERENCE OPERATORS OF SECOND ORDER

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Abstract. Study of the spectral analysis of non-selfadjoint Sturm Liouville operators (SLO) with continuous and point spectrum was begun by Naimark [11]. He proved that the spectrum of non-selfadjoint SLO consists of the continuous spectrum, the eigenvalues, and the spectral singularities. The spectral singularities are poles of the kernel of the resolvent and are also imbedded in the continuous spectrum, but they are not eigenvalues. In [10] the effect of the spectral singularities in the spectral expansion of SLO in terms of the principal vectors was considered. Some problems of spectral theory of differential and difference operators with spectral singularities were also studied in[1- 9].

In this paper we investigate the spectrum of the non-selfadjoint difference operator L generated in $l_2(\mathbb{N})$ by the difference expression

$$l(y)_n = a_{n-1}y_{n-1} + b_n y_n + a_n y_n;\ n \in \mathbb{N};\ a_0 = 1;$$

and the boundary condition $y_0 = 0$; where $\{a_n\}_{n \in \mathbb{N}}$ and $\{b_n\}_{n \in \mathbb{N}}$ are complex sequences. We prove that L has the continuous sprctrum, filling the segment $[-2; +2]$; a finite number of eigenvalues and spectral singularities with finite multiplicities if

$$\sum_{n=1}^{\infty} e^{\varepsilon\sqrt{n}}(|1-a_n| + |b_n|) < \infty;\ \varepsilon > 0;$$

holds. The results about the spectrum of L are applied to the non-selfadjoint Jacobi matrices and discrete Schrödinger operators. Moreover we derived the spectral expansion of L in terms of the principal vectors, taking into account the spectral singularities.

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