The Set of Completely Positive Maps and its Geometric Properties

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Abstract

Let $C$ be a closed convex set in a topological vector space $(X, \tau)$. An element $\omega \in C$ is said to be an exposed point if there exists a $\tau$-continuous linear functional $f$ on $X$ such that $Re f(x) < Re f(\omega)$ for all $x \in C \setminus \{\omega\}$. For a unital $C^*$-algebra $A$ and complex Hilbert space $H$, extremal structure of the set of all completely positive linear maps $CP(A, H)$ of $A$ into $B(H)$, where $B(H)$ denotes the $C^*$-algebra of all bounded linear operators on $H$ was determined several years ago by Arveson [Acta Math. 123 (1969), 141-224]. In this note we study geometric structure of these sets in terms of exposed points.