Rate Independent Evolution Variational Inequality with a Non linear Elliptic Part

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Abstract

In this paper we study the properties of the following evolution variational inequality with a non linear elliptic part. Let A be a non linear operator on H into its dual H^* find $w : [0,T] \to H, w(0) = 0$ such that for almost all $t \; \varepsilon (0,T), \dot{w}(t) \; \varepsilon K$ and

$$\langle Aw(t), z - \dot{w}(t)\rangle \geq \langle l(t), \quad z - \dot{w}(t)\rangle, \forall \;\; z \; \varepsilon \; K, \quad \dot{w}(t) \; \varepsilon \; K \; , w(0) = 0. \tag{1}$$

A special case of this inequality, where bounded linear operator A is considered, has been studied by Han, Reddy and Schrolder [SIAM J. Num. Anal. 34(1997), 143-177].

Theorem 1. Let H be a Hilbert Space, $K \subseteq H, \; K \neq \phi$, closed and convex cone; $A : \;\; H \to H^*$ be monotone coercive and Lipschitz continous. Furthermore, let $l \; \varepsilon W^{1,2}(0,T;H^*)$ with $l(0) = 0$, then there exists at least one $w \; \varepsilon \; W^{1,2}(0,T;H^*)$ satisfying (1), w is unique if, in addition, A is strictly monotonic.

Theorem 2. Under the assumption of Theorem 1, the solution of (1) , depends continuously on l, namely for $l_1, \; l_2 \; \varepsilon \; W^{1,2}(0,T;H^*)$ with $l_1(0) \; = \; l_2(0)$, the

corresponding solutions $w_1,\ w_2$ satisfy

$$\| w_1 - w_2 \|_{L^\infty(0,T;H)} \leq C(\| l_1 - l_2 \|_{L^\infty(0,T;H^*)} + \| \dot{l}_1 - \dot{l}_2 \|_{L^1(0,T;H^*)}) \tag{2}$$

Theorem 3. Let P_h denote the approximate problem of (1) and let w_1^h and w_2^h be two solutions of P_h, then

$$\| w_1^h - w_2^h \|_{L^\infty(0,T;H^*)} \leq C(\| l_1 - l_2 \|_{L^\infty(0,T;H^*)} + \| \dot{l}_1 - \dot{l}_2 \|_{L^1(0,T;H^*)}) \tag{3}$$

$$\| w(t) - w^h(t) \|_{L^\infty(0,T;H^*)} \leq C\ inf\ \{\| w - z^h \|_{L^1(0,T;H)}^{\frac{1}{2}}\} \tag{4}$$

where inf is taken for all $z^h\ \varepsilon\ L_2(0,T;K^h)$, and K^h is a non empty closed and convex cone.