GENERAL INCLUSION RELATIONS FOR ABSOLUTE SUMMABILITY

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In a recent paper the author [?] obtained necessary conditions for a series summable $|A_k|$, $1 < k \leq s < \infty$, to imply that the series is summable $|B_s|$ where $A$ and $B$ are lower triangular matrices. In this paper we obtain sufficient conditions for a series summable $|A_k|$, $1 < k \leq s < \infty$, to imply that the series is summable $|B_s|$. Using these results we obtain a number of corollaries.

Let $T$ be a lower triangular matrix, $\{s_n\}$ a sequence. Then

$$ T_n := \sum_{\nu=0}^{n} t_{n\nu} s_\nu. $$

A series $\sum a_n$ is said to be summable $|T|_k$, $k \geq 1$ if

$$ \sum_{n=1}^{\infty} n^{k-1} |T_n - T_{n-1}|^k < \infty. $$

(1)

We may associate with $T$ two lower triangular matrices $\mathbf{T}$ and $\hat{T}$ as follows:

$$ \hat{t}_{n\nu} = \sum_{r=\nu}^{n} t_{nr}, \quad n, \nu = 0, 1, 2, \ldots, $$

and

$$ \hat{t}_{n\nu} = \hat{t}_{n\nu} - \hat{t}_{n-1,\nu}, \quad n = 1, 2, 3, \ldots. $$

With $s_n := \sum_{i=0}^{n} a_i$. 

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\[ y_n := \sum_{i=0}^{n} t_{ni} s_i = \sum_{i=0}^{n} t_{ni} \sum_{\nu=0}^{i} a_{\nu} \]
\[ = \sum_{\nu=0}^{n} a_{\nu} \sum_{i=\nu}^{n} t_{ni} = \sum_{\nu=0}^{n} \tilde{t}_{n\nu} a_{\nu} \]

and

\[ Y_n := y_n - y_{n-1} = \sum_{\nu=0}^{n} (\tilde{t}_{n\nu} - \tilde{t}_{n-1,\nu} a_{\nu}) = \sum_{\nu=0}^{n} \tilde{t}_{n\nu} a_{\nu}. \]

We shall call \( T \) a triangle if \( T \) is lower triangular and \( t_{nn} \neq 0 \) for each \( n \). The notation \( \Delta_{\nu} \hat{a}_{n\nu} \) means \( \hat{a}_{n\nu} - \hat{a}_{n,\nu+1} \).

**Theorem 1.** Let \( 1 < k \leq s < \infty \). Let \( A \) and \( B \) be triangles satisfying

(i) \( |b_{nn}| \leq O\left(\nu^{1/s-1/k}\right) \),

(ii) \( (n|X_n|)^{s-k} = O(1) \),

(iii) \( |a_{nn} - a_{n+1,n}| = O(|a_{nn}a_{n+1,n+1}|) \),

(iv) \( \sum_{\nu=0}^{n-1} |\Delta_{\nu} (\hat{b}_{n\nu})| = O(|b_{nn}|) \),

(v) \( \sum_{n=\nu+1}^{\infty} (n|b_{nn}|)^{s-1} |\Delta_{\nu} (\hat{b}_{n\nu})| = O(\nu^{s-1}|b_{nu}|^s) \),

(vi) \( \sum_{\nu=0}^{\infty} |b_{nu}|^s |\hat{b}_{n,\nu+1}| = O(|b_{nn}|) \),

(vii) \( \sum_{n=\nu+1}^{\infty} (n|b_{nn}|)^{s-1} |\hat{b}_{n,\nu+1}| = O((\nu|b_{nu}|)^{s-1}) \),

and

(viii) \( \sum_{n=1}^{\infty} n^{s-1} \left| \sum_{\nu=2}^{n} \hat{b}_{nu} \sum_{i=0}^{\nu-2} \hat{a}'_{\nu i} X_i \right|^s = O(1) \).

Then if \( \sum a_n \) is summable \( |A|_k \), it is summable \( |B|_s \).

**References**


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