GENERAL INCLUSION RELATIONS FOR ABSOLUTE SUMMABILITY

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In a recent paper the author [?] obtained necessary conditions for a series summable $|A_k|$, $1 < k \leq s < \infty$, to imply that the series is summable $|B_s|$ where A and B are lower triangular matrices. In this paper we obtain sufficient conditions for a series summable $|A_k|$, $1 < k \leq s < \infty$, to imply that the series is summable $|B_s|$. Using these results we obtain a number of corollaries.

Let T be a lower triangular matrix, $\{s_n\}$ a sequence. Then

$$T_n := \sum_{\nu=0}^{n} t_{n\nu} s_\nu.$$

A series $\sum a_n$ is said to be summable $|T|_k, k \geq 1$ if

$$\sum_{n=1}^{\infty} n^{k-1} |T_n - T_{n-1}|^k < \infty. \tag{1}$$

We may associate with T two lower triangular matrices $\overline{T}$ and $\hat{T}$ as follows:

$$\bar{t}_{n\nu} = \sum_{r=\nu}^{n} t_{nr}, \qquad n, \nu = 0, 1, 2, \ldots,$$

and

$$\hat{t}_{n\nu} = \bar{t}_{n\nu} - \bar{t}_{n-1,\nu}, \qquad n = 1, 2, 3, \ldots.$$

With $s_n := \sum_{i=0}^{n} a_i$.

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$$\begin{aligned} y_n &:= \sum_{i=0}^{n} t_{ni} s_i = \sum_{i=0}^{n} t_{ni} \sum_{\nu=0}^{i} a_\nu \\ &= \sum_{\nu=0}^{n} a_\nu \sum_{i=\nu}^{n} t_{ni} = \sum_{\nu=0}^{n} \bar{t}_{n\nu} a_\nu \end{aligned}$$

and

$$Y_n := y_n - y_{n-1} = \sum_{\nu=0}^{n} (\bar{t}_{n\nu} - \bar{t}_{n-1,\nu} a_\nu) = \sum_{\nu=0}^{n} \hat{t}_{n\nu} a_\nu. \tag{2}$$

We shall call T a triangle if T is lower triangular and $t_{nn} \neq 0$ for each n. The notation $\Delta_\nu \hat{a}_{n\nu}$ means $\hat{a}_{n\nu} - \hat{a}_{n,\nu+1}$.

Theorem 1. *Let $1 < k \leq s < \infty$. Let A and B be triangles satisfying*

(i) $\dfrac{|b_{nn}|}{|a_{nn}|} = O\left(\nu^{1/s-1/k}\right)$,

(ii) $(n|X_n|)^{s-k} = O(1)$,

(iii) $|a_{nn} - a_{n+1,n}| = O(|a_{nn} a_{n+1,n+1}|)$,

(iv) $\displaystyle\sum_{\nu=0}^{n-1} |\Delta_\nu(\hat{b}_{n\nu})| = O(|b_{nn}|)$,

(v) $\displaystyle\sum_{n=\nu+1}^{\infty} (n|b_{nn}|)^{s-1} |\Delta_\nu(\hat{b}_{n\nu})| = O(\nu^{s-1}|b_{\nu\nu}|^s)$,

(vi) $\displaystyle\sum_{\nu=0}^{n-1} |b_{\nu\nu}||\hat{b}_{n,\nu+1}| = O(|b_{nn}|)$,

(vii) $\displaystyle\sum_{n=\nu+1}^{\infty} (n|b_{nn}|)^{s-1} |\hat{b}_{n,\nu+1}| = O((\nu|b_{\nu\nu}|)^{s-1})$,

and

(viii) $\displaystyle\sum_{n=1}^{\infty} n^{s-1} \left| \sum_{\nu=2}^{n} \hat{b}_{n\nu} \sum_{i=0}^{\nu-2} \hat{a}'_{\nu i} X_i \right|^s = O(1)$.

Then if $\sum a_n$ is summable $|A|_k$, it is summable $|B|_s$.

References

[1] Ekrem Savaş, Necessary conditions for inclusion relations for absolute summability , to appear in Appl. Math. Comp.

[2] B. E. Rhoades, Inclusion theorems for absolute matrix summability methods,*J. Math. Anal. Appl.***238**(1999), 82-90.
[3] B. E. Rhoades and Ekrem Savaş, On inclusion relations for absolute summability,*IJMMS*, **32(3)**(2002), 129-138.

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